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A Maritime Traffic Control Model for the Venezuela Navy

Salcedo Franco, Julian

Monterey, California. Naval Postgraduate School

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THESIS

A MARITIME TRAFFIC CONTROL MODEL FOR THE VENEZUELA NAVY

by

Julian Salcedo Franco
September 1997

Thesis Advisor:	Kneale T. Marshall
Second Reader:	Patricia Jacobs

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The research area of this thesis is an operational issue of the Venezuelan Navy. The specific area of research is the development of a Maritime Traffic Control Model that guarantees the efficient accomplishment of surveillance and protection of the territorial sea. The principal task is reducing the number of potential illegal shipments, for example drugs, toxic waste, or other outlaw activities in the Venezuelan sea.

Currently, several operational activities are executed as a response against illegal shipments. However, these operational activities require many resources and a considerable amount of time. These operational tasks depend primarily on Intelligence efforts that represent a high financial cost and additional risky actions. For these reasons, the successful execution of the maritime control mission requires more dynamic and efficient approaches to maximize operational benefits.

This thesis develops and uses a stochastic decision making model, to analyze and set up inspection or interdiction operations in those areas whose geographic features represent closed transit areas for navigation. This decision model should be an important aid for the execution of Maritime Traffic Control Operations in closed maritime areas in the Caribbean Sea under the sovereignty of Venezuela.

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A MARITIME TRAFFIC CONTROL MODEL FOR THE VENEZUELA NAVY

Julian Salcedo Franco
Commander, Venezuela Navy
BS, Venezuela Naval Academy, 1981

Submitted in partial fulfillment
of the requirements for the degree of

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September 1997

ABSTRACT

The research area of this thesis is an operational issue of the Venezuelan Navy. The specific area of research is the development of a Maritime Traffic Control Model that guarantees the efficient accomplishment of surveillance and protection of the territorial sea. The principal task is reducing the number of potential illegal shipments, for example drugs, toxic waste, or other outlaw activities in the Venezuelan sea.

Currently, several operational activities are executed as a response against illegal shipments. However, these operational activities require many resources and a considerable amount of time. These operational tasks depend primarily on intelligence efforts that represent a high financial cost and additional risky actions. For these reasons, the successful execution of the maritime control mission requires more dynamic and efficient approaches to maximize operational benefits.

One solution for this problem is the development of a stochastic decision making model, to analyze and set up inspection or interdiction operations in those areas whose geographic features represent closed transit areas for navigation. This decision model should be an important aid for the execution of maritime traffic control operations in closed maritime areas in the Caribbean Sea under the sovereignty of Venezuela.

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LIST OF SYMBOLS

t	period of time unit (week, month, etc.).
ℓ	period length (hours).
I	time to inspect a ship.(hours).
m	number of areas to be patrolled, indexed $j = 1,2,3....m$.
λ_j	average number of outside ships coming to harbor through area j during a time period of length ℓ.
$k_j(t)$	number of "bad" ships found in area j in a period t
R_j	updated estimate of the fraction of illegal ships in area j after an inspection is carried out.
$X_j(t)$	fraction of ships entering area j that are "bad" (have an illegal shipment on board) during time period t.
$A_j(t)$	number of assets assigned to area j in period t. An asset consists of a pair of ships.
n	total number of assets that are available for assignment.
Cap_j	number of ships that an asset can inspected per period of time in area j.
$d_j(t)$	number of ships inspected in area j in period t.
ε	odds ratio confidence level
$\mu_j(t)$	updated mean of the proportion of ships carrying illegal cargo in area j after all inspections are carried out in period t.
$\sigma_j^2(t)$	updated variance of the proportion of ships carrying illegal cargo in area j after all inspections are carried out in period t.
$CV_j(t)$	coefficient of variation of the proportion of ships carrying illegal cargo in area j after inspections in period t
φ	coefficient of variation confidence level.

EXECUTIVE SUMMARY

The research area of this thesis is an operational issue of the Venezuelan Navy. The specific area of research is to develop a Maritime Traffic Control Model that guarantees the efficient accomplishment of surveillance and protection of the Territorial Sea. The principal task is reducing the number of potential illegal shipments, for example drugs, toxic waste, or other outlaw activities in the Venezuelan sea.

Currently, several operational activities are executed as a response against this threat. However, these operational activities require many resources and a considerable amount of time with negative tradeoffs. These operational tasks depend primarily on Intelligence efforts that represent a high financial cost. For these reasons the successful execution of the Maritime Control Mission requires more dynamic and efficient approaches to maximize operational benefits.

One solution for this problem is the development of a stochastic decision making model, to analyze and set up inspection or interdiction operations in those areas whose geographic features represent closed transit areas for navigation. This Decision model should be an important aid for the execution of Maritime Traffic Control Operations in closed maritime areas in the Caribbean Sea under the sovereignty of Venezuela.

The main mission of the Venezuelan Navy, during peace, is to guarantee the observance of the national laws and international rules inside its territorial waters. One way to accomplish this mission is the permanent surveillance and traffic control for vessels in its waters. Venezuela is the neighbor of several countries in the Caribbean Sea. By its geographical position, Venezuela is one of the principal maritime routes and sometimes serves as an intermediate ‘base” for the entire maritime traffic in South America. Also, because of its tropical and calm weather, Venezuela has a heavy tourist maritime traffic, mainly from other Caribbean colonies or islands. All these factors create a maritime environment that is often difficult to control and to enforce the observance of laws. Currently, illegal activities such as drug dealing, transport of chemical waste and other illegal activities are some of the new persistent problems. Their presence requires more forces and a different approach from that of conventional warfare to defeat them.

This view makes such operations other than warfare missions a daunting duty. To face this problem, many questions arise: 1) What is the approximate number of ships that come in or come out from Venezuelan harbors? 2) What is the area with the biggest maritime traffic? 3) Given the current number of available resources, how we should allocate them to execute the desired maritime control in an efficient way? 4) Using the geographical characteristics of certain coast areas (Natural Navigation Channels), is it possibly to carry out control policies and inspect the maritime traffic, according to the current "intelligence" functions?

This thesis develops the required decision model to assist in the process of allocation of inspection assets to areas.

This study formulates a decision model with stochastic events. Its application exploits the geographical aspect of the Venezuela Coast and the current maritime scenario with potential threats like drug dealers and other illegal commercial activities. A model provides an analytical procedure to determine the best allocation of assets for the execution of a maritime control mission in closed areas. Asset allocation is based on the fraction of ships that are carrying illegal cargo in each area j during a time period t. This model uses forecast information or 'intelligence" to deploy such assets according to a Bayesian estimation of the illegal fraction. Few quantitative data related to this issue are available, which makes difficult the use of classical statistical methods.

This thesis should form the introduction document for future studies in this type of operational requirement, using a probabilistic approach and a decision model to measure the results of the use of policies.

The model estimates the proportion or fraction of ships carrying illegal cargo which heads to harbor through natural straits or channels. Using the available 'intelligence" as input to generate an initial estimate of this fraction, the model formulates a decision making process as a guide to the planner for asset allocation. The initial allocation decision is based upon a weighting using the initial estimated mean of the fraction of illegal ships with illegal cargo, μ_j. Those areas to be inspected with higher fractions will receive more assets. The model demonstrates that if the decision maker decides to get a precise estimation of μ_j by carrying out several inspection with a fixed

number of assets in each area, then he/she can maximize the number of illegal ships found with little change in the asset allocation. On the contrary, when the planner decides to make inspections only with the prior information from intelligence, he/she may reallocate assets more frequently, and more inspections are needed to get an estimated μ_j value with a desired precision. Even though this thesis keeps separate the logistics considerations related to the assets allocation, the fact remains that reducing the uncertainty of the μ_j value should be helpful in minimizing the complexity of the allocation problem.

This thesis involves the intelligence as a quantitative input into the decision making process. The Bayesian approach makes possible the use of "subjective" information into the analytical model. The model is used to demonstrate that "intelligence" can be translated as an important estimator in this stochastic analysis.

I. INTRODUCTION

A. BACKGROUND AND THEORETICAL FRAMEWORK

This purpose of this study is to improve current Venezuelan Navy maritime control policies. In the accomplishment of its mission, the Venezuelan Navy is updating and creating new procedures and tactics to minimize the increasing number of illegal shipping activities in the Venezuelan territorial sea and interior waters. Also, financial and war material constraints require that more efficient methods are needed to execute this type of task and to ensure the achievement of peace and observance of the National Laws.

B. CURRENT SITUATION

The current Venezuela Maritime Control Doctrine is designed mainly to protect and to place under surveillance the Maritime traffic in a potential war situation. Though some procedures may be applied during peace situations, the doctrine's war application uses navigational restrictions. The current day-to-day maritime threats occur in open and unrestricted navigation areas. This means expeditious analysis and more efficient plans of actions are required. These plans demand a large amount of information related to the maritime situation in certain areas or regions. Unfortunately, this information or 'intelligence" should be updated very quickly, given the interaction of lawless actions and the dynamic nature of the Caribbean area. Therefore operational procedures need to be highly flexible and quickly set, due to the temporal value of the 'Intelligence." The value of intelligence decreases rapidly with time.

The contemporary threats require an immediate response. For example: an increasing and big menace is drug trafficking. Drug traffickers use every kind of craft to transport drugs. They use any route at anytime to guarantee the successful arrival of illegal goods. Their dangerous procedures and tactics require large numbers of maritime control resources.

A constantly changing situation requires the ability to make swift decisions. Every day, dozens of ships navigate in Venezuelan waters. The Venezuelan Maritime area includes a large coast with many accessible sites. Several Sovereign State Islands lie close to Venezuela in their own territorial seas. This fact makes surveillance more difficult due to legal restrictions. A potential tactic of using natural channels (straits, rivers, river mouths) might simplify the development of ship inspection plans. These natural channels usually create "rows" of ships coming into or out of Venezuelan harbors. Also, Venezuela owns several small islands, which form a set of "narrow corridors." The use of focal patrol groups in these areas may allow a more efficient way to concentrate resources with subsequent favorable results. This situation is shown in Figure 1.

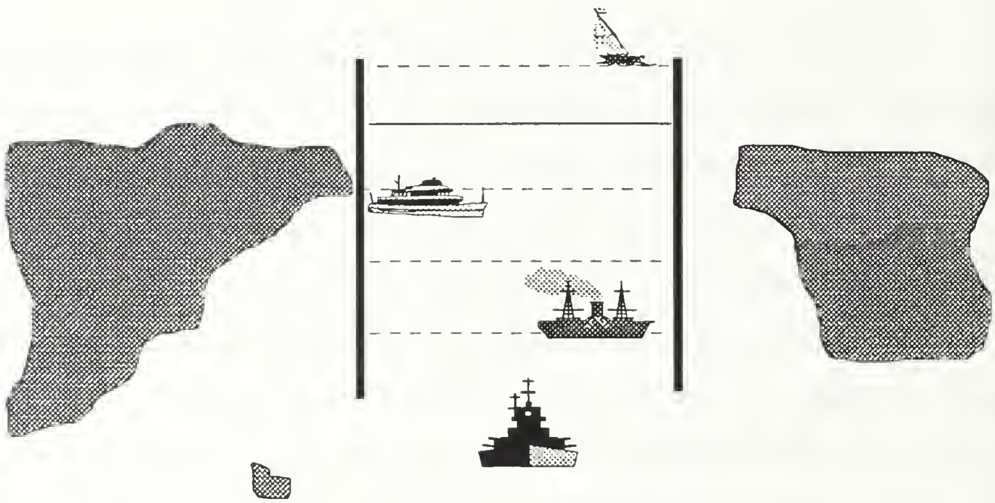

Figure 1. A ship inspection plan in a Natural channel

This thesis develops the required decision model to assist in allocating patrol groups to areas to carry out ship inspections.

C. THESIS GOALS AND OUTLINE

This thesis formulates a decision model with stochastic events. The model exploits the geographical features of the Venezuelan coast and the current maritime scenario with its potential threats of drug dealers and other illegal commercial activities. This study responds to an operational requirement to support the Venezuelan Navy mission, which includes the enforcement of the national laws and the acquisition of the available means to provide enough capacity for this type of task.

This thesis is intended to form the introduction document for future studies of this type of operational requirement, using a probabilistic approach to decision models to assess the results of potential policies.

The model will have as input the number of ships traversing different natural channels during a time period, an assessment of the fraction of these ships carrying illegal cargo, the number of ships a patrol can inspect during a time period and an objective function. The output of the model will be a policy to allocate the finite number of patrols to the different natural channels.

Unfortunately no available data exist as to the current number of ships that come into or out of Venezuelan harbors. Many outsider recreational and fishing ships travel in these waters. They usually do not report their presence to the local authorities. However, tankers or merchant ships are regulated in a better way. However, their number is much smaller than the number of small ships described above.

In spite of the lack of data, a simulation can be used to assess measures of effectiveness for different policies. The decision model will specify the resource allocation, given its objective and the corresponding measures of effectiveness.

D. THE MARITIME CONTROL TRAFFIC PROBLEM IN THE CARIBBEAN SCENARIO

Currently the Venezuelan Navy executes maritime traffic control following traditional tactical procedures and deploys its forces in different areas along its coast line. The number of deployed ships corresponds to the availability of ships and is fixed. Sometimes, the planner is required to move ships from one area to another based on operational requirements, and the area losing the ships must sometimes wait for a long period of time to receive additional resources. Often during this time, many opportunities to intercept or deter illegal actions are missed, with substantial waste of time and resources. Further, constant operational activities compel tight execution schedules. The lack of a specified procedure to execute surveillance operations can divert resources toward activities more related to traditional operational tasks.

Nowadays, Venezuela and other nations have signed several agreements to control and coordinate efforts against illegal activities such as contraband and drug trafficking. The importance and exigency of this current issue implies the implementation of a tactical tool to support efforts against illegal activities, to monitor the observed results and to make the necessary decision in "real time".

II. FORMULATION OF THE DECISION MAKING MODEL

A. MODEL ASSUMPTIONS

1. Intelligence

A military operation should not be executed without prior knowledge of the probable scenario. To ensure success an efficient employment of resource information or "intelligence" is required to decide the best action, given the objective of the mission. In this study, it is assumed some intelligence is available to forecast the presence of illegal shipments in a given area or scenario.

2. Inspection Clearance

Any ship in or entering Venezuela waters can be examined. This assumption is stated to ensure the randomness of the selection of a ship for inspection.

3. Inspection Time and illegal shipment detection

To make the model formulation more tractable in this thesis, it is assumed that the time to inspect a ship is known and the detection of an illegal shipment onboard an inspected ship is certain. This is the most important assumption of this model because of its impact on the randomness of the inspection process and complexity of the analysis.

4. Number of required resources to inspect a ship

It is assumed that the minimum number of required resources to inspect a ship is two vessels. This is based upon prior experiences regarding safety and backup.

5. Asset's inspection capacity

A certain fixed asset's inspection capacity is considered for this analysis. Given that the inspection time is known and fixed, it is possible to calculate how many ships an asset can inspect during a given period of length ℓ.

6. Time to reallocate resources

One important issue is the time to reallocate resources, after an inspection mission is executed and the decision maker decides to move resources from area j to area k. This time is considered to be negligible; it is assumed there is enough time between the time of the decision to reallocate and the start of the new inspection mission.

7. Resources to be reallocated

Regardless of its initial position, any unit or ship can be reallocated to a new region or area. No logistic or operational constraints affect the desired resource's performance. Further, only an even number of ships will be repositioned due to the assumption that two ships are needed to make an inspection.

8. Ships to be inspected

For this investigation, it is assumed the probability of carrying an illegal cargo is the same for any ship randomly chosen in area j during a time period t. However, it is both possible and likely that the available "intelligence" suggests that some ships are more likely to be carrying illegal cargo than others. This consideration can be incorporated in further enrichments of this model. For example, a method for ranking the incoming ships and estimating the probability that a particular ship is transporting an illegal cargo could be created. Such a procedure would assist in determining which ship will be inspected and to maximize the expected number of illegal shipments detected.

9. Ships Arrival

For this study, the ships arrival process is assumed as follows: each ship heads to harbor independently the others. The number of ships that has arrived to harbor during a time period (t, t+1) is independent of the number of incoming ships in a time period (t-1,t). Further, the rate of incoming ships does not depend on the time of year or specific activities during the year; it is considered constant. The arrival process is characterized by a flow rate, λ_j; the expected number of incoming ships during a period length ℓ.

B. MODEL STRUCTURE

This model represents a periodic decision making process. For example, each period may consist of hours, days, a week, a month, etc. At the start of each period, the planner decides how many patrol assets he/she will employ and how they should be allocated to different areas. The allocation determines the maximum number of ships that can be inspected in each area. At the end of each period, the decision maker analyzes the results and decides to move assets from one area to another or to keep the same asset distribution. Figure 2 is used to clarify the time notation used in this study. Period t refers to the interval (t-1, t].

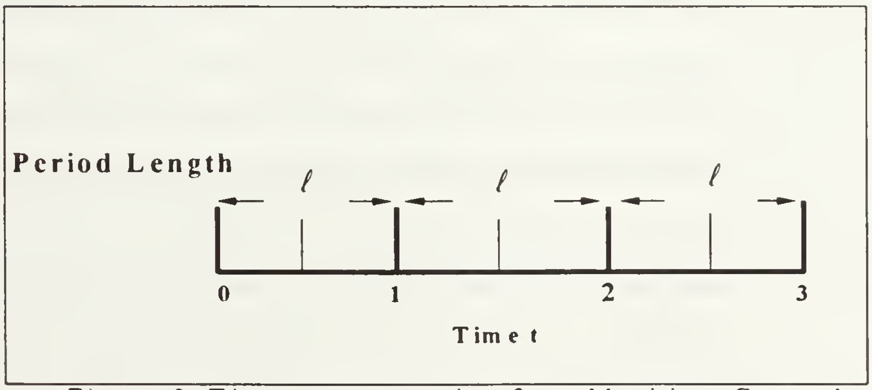

Figure 2. Time representation for a Maritime Control Mission

Inspection periods are of equal length ℓ, and periods are numbered sequentially starting at one. The index 0 refers to the starting information that is known before the first inspection starts.

1. Notation

The following notation is used to describe the elements or factors involved in the analysis:

t	index of time (see Figure 2).
ℓ	period length (hours). (see Figure 2).
I	time to inspect a ship.(hours).
m	number of areas to be patrolled, indexed $j = 1,2,3....m$.
λ_j	Flow rate of outside ships coming to harbor through area j during a time period of length ℓ,
X_j	Fraction of ships entering area j that are 'bad" (have an illegal shipment on board). This fraction is unknown and so is treated as a random variable.
n	Total number of assets that are available for assignment.
$A_j(t)$	number of assets assigned to area j in period t. An asset consists of a pair of ships. This is a decision variable. Also, we must have that $\sum_{j=1}^{m} A_j(t) \le n$ for every t.
Cap_j	The number of ships that an asset can inspect per time period in area j.
$d_j(t)$	number of ships to be inspected in area j in period t. $d_j(t)_{max} = Cap_j A_j(t)$.
$k_j(t)$	number of 'bad" ships found in area j in period t.
$\mu_j(t)$	Updated mean of the proportion of ships in area j that are carrying illegal cargo after all inspections are carried out in period t.

$\sigma_j^2(t)$	Updated variance of the proportion of ships in area j that are carrying illegal cargo after all inspections are carried out in period t.

2. The Objective Functions

In a real scenario it is difficult to inspect every ship coming to port. First, to inspect a ship at sea is a very hard task with many risks, and is usually highly time consuming. Second, to inspect all ships would demand a huge number of resources and many hours of operations. These considerations are just some of the constraints present in the execution of such missions.

Therefore, the problem is how to choose the number of assets, $A_j(t)$, to be employed in area j during an inspection period t, to achieve one or more of the following objectives:

A) Reduce the uncertainty on X_j.

B) Maximize the expected number of 'bad" ships inspected in area j, given the limit on inspection resources.

C) Reduce the expected value of X_j through deterrance.

In analyzing these goals, there is an order of priority. The goal A) must come first because the proportion of illegal ships in each area must be known as accurately as possible before goals B) or C) can be accomplished. This priority order is followed. It is assumed that there is no change in the habitual 'tactics" or shipments procedures executed by 'bad" ships during successive inspection periods. Once goal A is achieved, the decision maker can decide how many assets should be assigned in each area j to maximize the expected number of 'bad" ships intercepted (Objective B). Later, the correct asset allocation will induce the reduction of the expected value of X_j, due to the permanent law enforcement through deterrance.(Objective C).

3. The Decision Algorithm

In the planning of resources allocation, there is an 'intelligence" input that predicts the percentage or fraction of ships that carry illegal cargo into each area j. This information is modeled using a Beta random variable, whose general pdf is :

$$f(p;\ (\alpha_j(t), \beta_j(t))\ ) = \frac{\Gamma(\alpha_j(t)+\beta_j(t))}{\Gamma(\alpha_j(t)) * \Gamma(\beta_j(t))} p^{\alpha_j(t)-1}(1-p)^{\beta_j(t)-1}, \text{ for } 0 \le p \le 1.$$

The steps of the procedure are as follows:

(i) The fraction of ships entering area j in a time period that contains illegal cargo is assumed to have a Beta distribution with parameters $\alpha_j(0)$ and $\beta_j(0)$. The intelligence information is assumed to provide:

$\mu_j(0)$ = initial estimate of the mean fraction of ships carrying illegal shipments through area j,

$\sigma^2_j(0)$ = initial estimate of the variance of the fraction of ships carrying illegal shipments through area j.

The mean of a beta distribution with parameters α and β is

$$\mu = \frac{\alpha}{\alpha+\beta}, \tag{2.1}$$

and its variance is

$$\sigma^2 = \frac{\alpha * \beta}{(\alpha+\beta)^2 * (\alpha+\beta+1)} \tag{2.2}$$

Thus, the initial parameter values $\alpha_j(0)$ and $\beta_j(0)$ can be obtained as follows:

$$\alpha_j(0) = \frac{\mu^2_j(0) * (1-\mu_j(0)) - \mu_j(0) * \sigma^2_j(0)}{\sigma^2_j(0)}. \tag{2.3}$$

$$\beta_j(0) = \frac{\alpha_j(0) * (1-\mu_j(0))}{\mu_j(0)}, \qquad j = 1,2...,m. \tag{2.4}$$

These are the parameters of the prior beta distribution for area j, given the forecast or 'intelligence". Equations (2.3) and (2.4) are used frequently in the remainder of the thesis. When used for some period t, the "0"s in these equations are replaced by "t"'s.

(ii) Having determined $\alpha_j(0)$ and $\beta_j(0)$, the planner chooses the number of assets to be deployed for the first inspection period in each area j, $A_j(1)$. The number of assets deployed is determined by the following relationship:

$$A_j(1) = n\left(\frac{\dfrac{\lambda_j \mu_j(0)}{\text{Cap}_j}}{\sum_{k=1}^{m} \dfrac{\lambda_k \mu_k(0)}{\text{Cap}_k}}\right), j = 1,2,\ldots,m. \tag{2.5}$$

$A_j(1)$ is the closest integer if (2.5) does not result in an integer.

(iii) After completion of the first inspection mission, the planner obtains the numbers of "bad" ships found in the first period, $\{k_j(1)\}$. It is assumed that each $k_j(1)$ is a sample from a binomial distribution with parameters $d_j(1)$, number of ships inspected in area j, and probability of success, $\mu_j(0)$. This "new" information is employed to update the beta distribution parameters for every area. The "posterior" distribution is also beta with parameters:

$$\alpha_j(1) = \alpha_j(0) + k_j(1), \tag{2.6.a}$$

$$\beta_j(1) = \beta_j(0) + d_j(1) - k_j(1). \tag{2.6.b}$$

From these, the updated values of the mean and variance of the proportion of "bad" ships in region j, $\mu_j(1)$ and $\sigma^2{}_j(1)$, are determined using Equations (2.1) and (2.2).

(iv) A test is made to see whether to end the algorithm or to return to step (ii). The decision criterion used will depend on the objective function and is discussed in the next section.

C. ASSETS ALLOCATION AND DECISION CRITERION

The decision making cycle is repeated until the decision maker's established objective has been achieved. Depending on the inspection mission's objective, the results will be tested, according to the following procedures:

1. Objective: Reduce the uncertainty of X_j.

In this case, the decision maker wants to get a more accurate value of the fraction of 'bad" ships in each area. To accomplish that purpose, he/she decides on an initial allocation using (2.5) and then continues that allocation.

After the t^{th} inspection period is completed, a coefficient of variation, $CV_j(t)$, is used as a statistic to measure the variability or relative dispersion of the observed proportion of illegal ships found in each area. $CV_j(t)$ is defined as:

$$CV_j(t) = \frac{\sigma_j(t)}{\mu_j(t)}. \tag{2.7}$$

Since the proportion of illegal ships after t inspection periods has a beta distribution with parameters $\alpha_j(t)$ and $\beta_j(t)$, substitution of Equations (2.1), (2.2) and (2.6.a,b) into (2.7) results in the coefficient of variation for the Beta distribution:

$$CV_j(t) = \sqrt{\frac{\beta_j(t-1) + d_j(t) - k_j(t)}{[\alpha_j(t-1) + \beta_j(t-1) + d_j(t) + 1]*[\alpha_j(t-1) + k_j(t)]}} \tag{2.8}$$

Once, the $CV_j(t)$ value is under a given limit, φ, the decision maker accepts the updated mean of the proportion of illegal ships for this time period .t, $\mu_j(t)$, as the actual fraction of illegal ships. How to choose the value of φ is discussed in Chapter III.

2. Objective: Maximize the number of illegal ships found in each area j.

After observing the results of the t^{th} inspection period the decision maker establishes a confidence interval for the odds ratio value ε to decide how to reallocate assets. Denote the lower and upper confidence limits by LCL and UCL respectively.

For any area j if $\frac{P(X_j > \text{LCL})}{P(X_j < \text{LCL})} < \varepsilon$, the decision maker will move resources from area j to other areas, so as to decrease $A_j(t+1)$.

For any area j if $\dfrac{P(X_j > \text{UCL})}{P(X_j < \text{UCL})} > \varepsilon$, the decision maker will move resources from other areas to area j, so as to increase A_j (t+1). The appropriate value of ε to be used is explained in detail in Chapter III as is the choice of LCL and UCL.

D. THE ASSESSMENT OF THE PRIOR INFORMATION

The use of Bayes' theorem gives an excellent opportunity to exploit all the information related to this problem, regardless of its structure or numerical representation. However, it is necessary to be careful in the selection of the "prior" intelligence as input for the initial assets allocation. First, unless there is strong evidence of the accuracy of the intelligence source, the planner should assume a large standard deviation $\sigma_j(0)$ for the fraction of ships containing illegal cargo in region j. A large standard deviation will allow the inspection results in the first period to change the parameters of the beta distribution considerably, allowing the estimated value of $\mu_j(1)$ to be much different than the initial estimate $\mu_j(0)$. In most, but not all cases, the posterior standard deviation will be smaller than the prior standard deviation if many "bad" shipments are found. This is intuitively reasonable, since the number of ships to be inspected, $d_j(1)$, will provide new information, and increased information should reduce the uncertainty about the fraction of ships that carry illegal shipments, X_j. Thus, it might be expected that additional information would reduce the standard deviation.

To illustrate this point, an estimated forecast or intelligence is used to observe the sensitivity of the mean $\mu_j(1)$ to variations in the prior information. In this example, a mean, $\mu_j(0)$ = 0.16 and a small standard deviation, $\sigma_j(0)$ = 0.06 for X_j are assumed, which give $\alpha_j(0)$ =5.8 and $\beta_j(0)$ = 30.5 using Equations (2.3) and (2.4).

Assume that an inspection of 12 ships is made (i.e., $d_j(1)=12$). The beta posterior parameters are calculated assuming various values of the number of inspected ships carrying illegal shipments during a time period t, $k_j(t)$, and are shown in the Table 1.

	# INSPECTED SHIPS ($d_j(t)$)=			12
	BETA POSTERIOR VALUES			
# BADS (k_j)	$\alpha_j(1)$	$\beta_j(1)$	$\mu_j(1)$	$\sigma^2_j(1)$
0	5.8	42.5	0.120	0.00214
2	7.8	40.5	0.162	0.00275
4	9.8	38.5	0.203	0.00328
6	11.8	36.5	0.244	0.00374
8	13.8	34.5	0.286	0.00414

Table 1. Posterior Beta Distribution Parameters Values. Small Variance

Table 1 shows how the inspection results barely affect the present information, given the prior standard deviation. For instance, the difference between the prior mean and the posterior mean for eight bad ships found out of 12 ships total is relatively small. Therefore the decision maker might conclude that the inspection does not improve the information of the proportion of illegal ships. This point confirms the ideas expressed above; a prior distribution with a small standard deviation does not lead to a posterior distribution that reflects the inspection results. For example, if eight bads were found in 12 inspections, the raw fraction of bads suggests that the fraction of bads is more likely to be closer to $2/3$ than 0.28. The eight bads could be a rare event that happened to occur. But one would have to be very certain of the initial estimate of 0.16 in order to make this interpretation. Figure 3 displays the posterior beta pdf for each number of bad ships found.

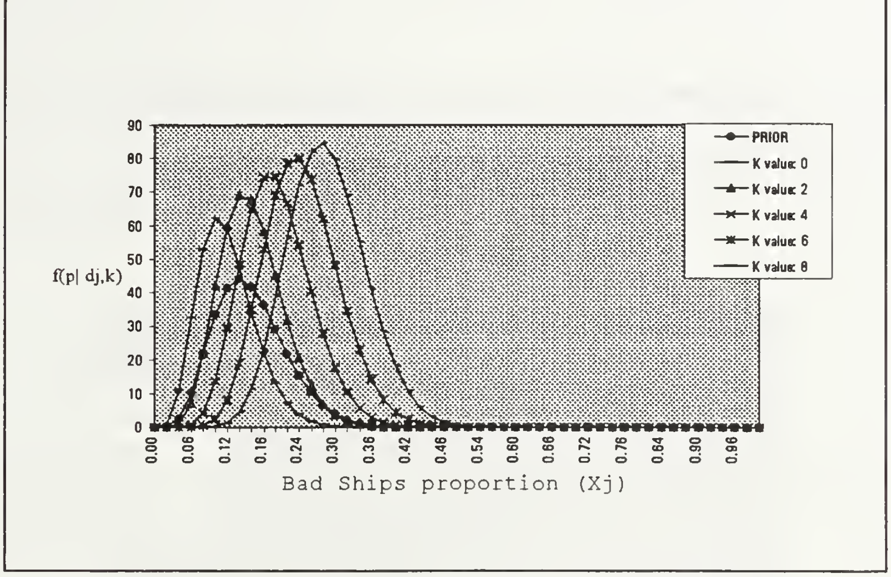

Figure 3. Posterior Beta pdf distributions. Small Standard deviation

On the other hand, a prior distribution with a large standard deviation will reflect easily any small change in the inspection results. This means that the "prior information" is more sensitive to the inspection results; this indicates the need for inspection and gives to the decision maker a better perception of the new information and the value of X_j.

In this example, a large prior standard deviation $\sigma_j(0) = 0.1$ is assumed and $\mu_j(0) = 0.16$ as before. The prior distribution with large standard deviation results in an unambiguous difference for each number of bad ships found during the inspection of 12 ships. Table 2 shows the inspection results with their respective posterior beta distribution parameters. In this case, there is a substantial difference between the means of the distinct inspection results. A larger prior standard deviation results in a easier discrimination of the inspection mission results due to the assets allocation.

	# INSPECTED SHIPS ($d_j(t)$)=			12
	BETA POSTERIOR VALUES			
# BADS (k_j)	$\alpha_j(1)$	$\beta_j(1)$	$\mu_j(1)$	$\sigma^2_j(1)$
0	2.0	22.4	0.081	0.00294
2	4.0	20.4	0.163	0.00537
4	6.0	18.4	0.245	0.00727
6	8.0	16.4	0.327	0.00865
8	10.0	14.4	0.409	0.00950

Table 2. Beta Posterior Distribution Parameters values. Large Variance.

Figure 4 shows this behavior graphically using the pdf for each Beta Posterior distribution values, given the number of bad ships found.

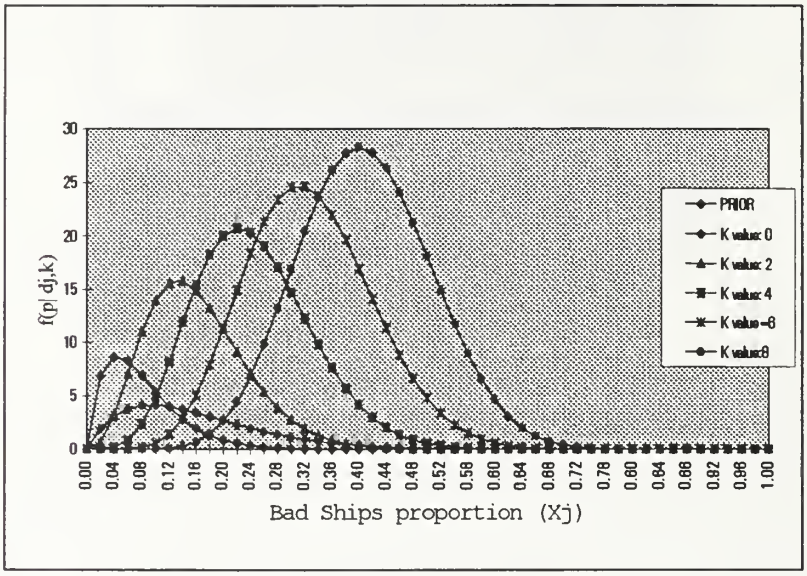

Figure 4. Posterior Beta Pdf Distributions. Large Standard deviation

Another useful consideration of Bayes' theorem is that the decision maker can wait and revise the estimated fraction after several inspection periods, using the combined inspection results. Given that inspections are independent each other, the same result is obtained whether the calculations are made after each inspection period or just once at the end of all of the inspection periods. This feature of Bayes' theorem is very important, since it reduces the number of applications of the theorem required to revise the proportion of illegal shipments on the basis of various inspections. Of course, the decision maker may want to revise after each inspection period, primarily because the resulting proportions may help to decide whether or not to execute another inspection.

III. INFERENCE AND MARGINAL ANALYSIS

A. THE ASSESSMENT OF THE "DECISION" REGION.

1. Objective A: Reduce the uncertainty of X_j

In Chapter II our first criterion for allocating resources is that the decision maker should attempt to estimate the fraction of ships carrying illegal shipments as precisely as possible, based upon the coefficient of variation, $CV_j(t)$. The coefficient of variation is compared to a number φ to evaluate the accuracy of $\mu_j(t)$, the estimated fraction of ships carrying illegal cargo in an area j in a period t.

Our criterion, $\frac{\sigma_j(t)}{\mu_j(t)} \leq \varphi$, can be reformulated as a function of the beta distribution parameters α and β. Rearrangement results in :

$$\sigma^2_j(t) \leq \varphi^2 \mu^2_j(t).$$

Equations (3.1) and (3.2) and substitution result in:

$$\frac{\alpha_j(t)\beta_j(t)}{(\alpha_j(t)+\beta_j(t))^2(\alpha_j(t)+\beta_j(t)+1)} \leq \varphi^2 \frac{(\alpha_j(t))^2}{(\alpha_j(t)+\beta_j(t))^2}$$

Hence,

$$\beta_j(t) \leq \varphi^2 \alpha_j(t)(\alpha_j(t)+\beta_j(t)+1).$$

When equality holds, substituting this expression for $\beta_j(t)$ in Equation (2.1) and simplifying, results in

$$\alpha_j(t) = \frac{1-\mu_j(t)-\mu_j(t)\varphi^2}{\varphi^2}. \tag{3.1}$$

Substituting (3.1) into Equation (2.4) yields:

$$\beta_j(t) = \frac{(1-\mu_j(t)-\mu_j(t)\varphi^2)}{\varphi^2}\frac{(1-\mu_j(t))}{\mu_j(t)}. \tag{3.2}$$

The α and β values at equality represent the posterior beta parameters for which the estimated proportion of illegal ships, $\mu_j(t)$, meets the accuracy expressed by φ, according to the coefficient of variation, CV_j. Figure 5 is used to illustrate this concept.

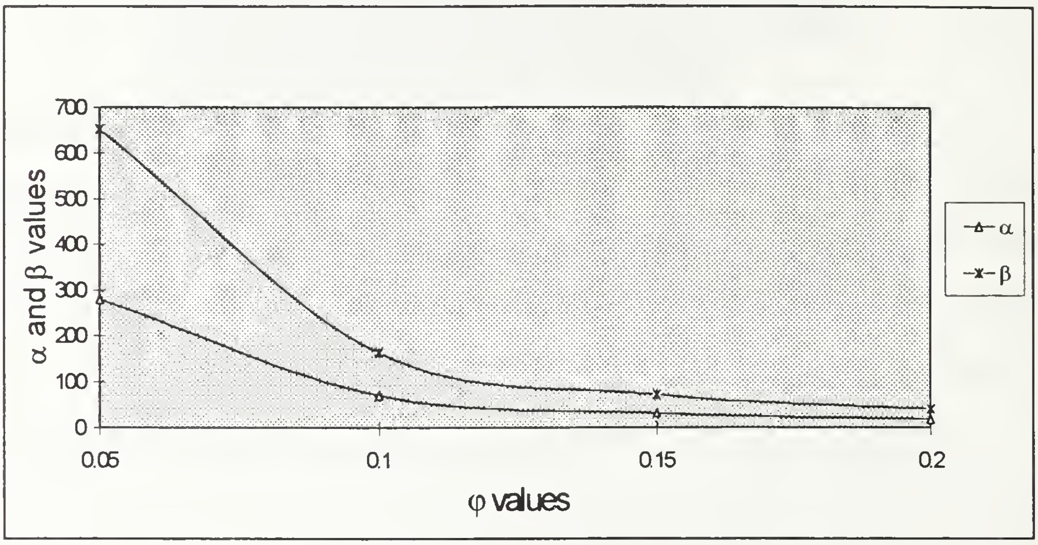

Figure 5. Posterior beta parameters as a function of φ

Figure 5 represents the α and β parameters as a function of the desired accuracy φ of the estimated mean of the proportion of illegal ships in area j, $\mu_j(t)$. In this case a $\mu_j(t)$ value target of 0.3 was employed. Note that the more accurate the estimated fraction of illegal shipments is required (i.e. the smaller the φ value), the higher α and β values are. Therefore, if the decision maker attempts to estimate the fraction of ships carrying illegal cargo in an area j with a high precision, then a bigger number of ships should be inspected to get a more accurate estimation of μ_j. This means a higher number of resources should be employed or more inspections should be carried out.

Following the above discussion, it is not clear that a constant value of φ can be used for a large range of $\mu_j(t)$ values, or what value of φ to use. Choosing a lower value of φ will ensure a lower variance on X_j relative to $\mu_j(t)$, but it will require more inspections (and hence time periods) to achieve the desired result.

Recall that after t inspection periods X_j has a beta distribution with mean $\mu_j(t)$, and standard deviation $\varphi\,\mu_j(t)$. Figure 6 shows the 95% credible interval for X_j for values of $\mu_j(t)$ between 0.1 and 0.3 when φ is 0.05 and 0.1. These credible intervals are calculated using the Excel Version 5 built-in BETAINV function.

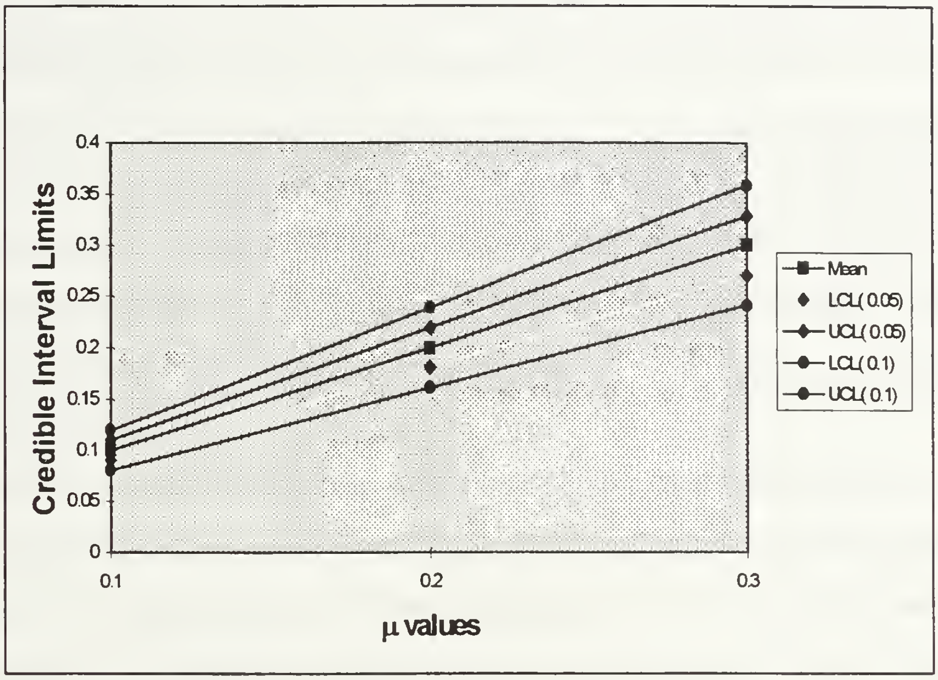

Figure 6. 95% Credible Interval for μ_j as a function of ϕ.

Note that the smaller the φ value, the smaller the credible interval. This credible interval should be interpreted as follows: there is a probability of .95 that the true fraction of ships with illegal cargo in an area j, X_j, takes a value between the interval limits. In the remainder of this thesis a φ value of 0.1 is used.

2. Objective B: Maximize the number of illegal incoming ships intercepted, $k_j(t)$.

In Chapter II, the odds ratio $\dfrac{P(X_j > x)}{P(X_j \leq x)}$ is suggested as the criterion to decide the reallocation of resources among the areas. Let ε represent the level by which to measure

the odds of the fraction of illegal ships, given the inspection results. To used this criterion we need to decide on a value of x as well as the value of ε.

Recall that $\mu_j(t)$ plays a central role in determining the allocation of assets using Equation (2.5). Therefore we should evaluate how the outcomes of the inspection in any period effect the estimation of $\mu_j(t)$. For this reason x will be set equal to $\mu_j(t)$.

Suppose objective A is complete in period t. Then X_j has a beta distribution with mean $\mu_j(t)$ and standard deviation $\varphi\,\mu_j(t)$. The odds that X_j exceeds $\mu_j(t)$ are $\frac{P(X_j \geq \mu_j(t))}{P(X_j < \mu_j(t))}$, and are shown in column 2 in Table 3. It is seen to be between 0.95 and 0.97 for every value of $\mu_j(t)$ between 0.02 and 0.3 when $\varphi = 0.1$. In period (t+1) the number of illegal ships found is denoted by $K_j(t+1)$. When this random variable takes on the value k_j our odds ratio, based on this new information, is $\frac{P(X_j \geq \mu_j(t)|K_j(t) = k_j)}{P(X_j < \mu_j(t)|K_j(t) = k_j)}$.

For small values of $\mu_j(t)$ we would not expect large numbers of illegal ships to be found. If a large value of k_j is found in period (t+1) it would indicate that $\mu_j(t+1)$ has increased from $\mu_j(t)$, and so more assets should be moved into area j. Similarly, a small value of k_j in (t+1) would indicate that $\mu_j(t+1)$ is smaller than $\mu_j(t)$, and so assets can be moved out of area j. Columns 3 through 10 in Table 3 show these (posterior) odds for values of $k_j(t+1)$ between 0 and 7 when $d_j(t+1)$ is 12.

Having set x equal to $\mu_j(t)$, the next problem is to decide on a value for ε. As has just been pointed out, a high or low conditional odds indicates a change in the value of $\mu_j(t)$. For reasons that are presented below we choose an upper value of ε to be 1.5 and a lower value of 0.67 when $\mu_j(t)$. is 0.16. Using these ε values in Table 3, if 5 or more illegal ships are found in period (t+1) we conclude that $\mu_j(t+1) > \mu_j(t)$ and use a new value together with other new values from other areas to reallocate assets using Equation (2.5). Similarly, if no illegal ships are found, we conclude that $\mu_j(t+1) < \mu_j(t)$ and reallocate accordingly. If between 1 and 4 illegal ships are found we conclude that $\mu_j(t+1)$

= μ_j (t). The calculation of μ_j (t+1) when it is considered different from μ_j (t) is shown in Section B below.

To justify the choices of 1.5 and 0.67 for upper and lower values of ε when μ_j (t). is 0.16, we further analyze the outcome of the inspection results in period (t+1). At the start of period (t+1), it is reasonably to assume that the number of illegal ships found in period (t+1) is a binomial random variable with parameters d_j(t+1) and μ_j (t). For the case when μ_j (t)= 0.16, $P(K_j(t+1) \geq 5)$ = 1-B(4; 12, 0.16) = 0.0310 and $P(K_j(t+1) \geq 4)$ = 1-B(3; 12, 0.16) = 0.111. Thus using a 5% cut-off value, we would claim μ_j (t+1) > μ_j (t) if $k_j(t+1) \geq 5$. This agrees with the value obtained using the ε value of 1.5 in Table 3. For other values of μ_j (t) in the range of 0.02 and 0.30, the critical levels of k_j(t+1) are approximately equal using ε values of 1.5 and 0.67 in Table 3, or cut-off values of 0.05 and 0.95 with the cumulative tail distribution of the binomial. These values are tabulated and presented in Appendix B.

This above example shows the procedure of calculating the required 95% odds ratio ε bounds. These bounds depend on the μ_j (t) value obtained in the completion of objective A and the number of ships to be inspected, d_j(t) Given that each area can have its own μ_j (t) value, its own asset inspection capacity and its own rate of incoming ships; the odds ratio bounds must be calculated for each area using the procedure described above.

B. INFERENCE AND DECISION FOR ASSETS ALLOCATION

To illustrate the preceding ideas, Table 3 is used to present the Odds values $\dfrac{P\left(X_j(t+1) \geq \mu_j(t) | K_j(t) = k_j\right)}{P\left(X_j(t+1) < \mu_j(t) | K_j(t) = k_j\right)}$ for different values of μ_j(t) from 0.02 to 0.30 and k_j from 0 to 7. Assume objective A) is completed at time t using a φ equal to 0.1, for some area j μ_j(t) = 0.16 , and so σ_j(t) =0.016. The probability that the true fraction of illegal ships, X_j(t), is greater than μ_j(t) is 0.49 so that the odds are 0.49/0.51= 0.96 (see column 2). Suppose that during inspection period (t+1) 6 of the 12 ships inspected are found to be carrying illegal goods. Using Equations (2.6.a) and (2.6.b) the new α_j(t+1) and β_j(t+1) are

89.974 and 446.865, respectively. Using Equation (2.1) the value of $\mu_j(t+1)$ is 0.167, and $P\big(X_j(t+1) > 0.16 | K_j(t+1) = 6\big)$ =0.674, so the odds are 0.674/0.328 = 2.07, a value above the upper bound ε of 1.5. The increase in odds from approximately 1 to 2 is a strong indication that the true fraction of illegal ships entering area j has increased, and that the finding of 6 illegal ships in 12 inspections can not be reasonably explained as simply a random occurrence.

			NUMBER OF SHIPS INSPECTED(d_j)=12						
	Prior		NUMBER ILLEGAL SHIPS DETECTED $k_j(t+1)$						
μ_j	Odds	$K_j=0$	$K_j=1$	$K_j=2$	$K_j=3$	$K_j=4$	$K_j=5$	$K_j=6$	$K_j=7$
0.02	0.95	0.91	1.07	1.26	1.49	1.75	2.06	2.43	2.86
0.04	0.95	0.86	1.04	1.22	1.44	1.70	2.01	2.38	2.83
0.06	0.95	0.84	1.00	1.16	1.40	1.66	1.97	2.33	2.76
0.08	0.95	0.81	0.96	1.14	1.35	1.61	1.92	2.28	2.73
0.10	0.95	0.77	0.92	1.10	1.31	1.56	1.87	2.23	2.67
0.12	0.96	0.74	0.86	1.06	1.27	1.51	1.82	2.16	2.62
0.14	0.96	0.70	0.84	1.01	1.22	1.47	1.76	2.12	2.57
0.16	0.96	0.67	0.81	0.97	1.17	1.42	1.71	2.07	2.51
0.16	0.96	0.63	0.77	0.93	1.13	1.37	1.66	2.01	2.45
0.20	0.96	0.60	0.73	0.89	1.08	1.32	1.60	1.96	2.39
0.22	0.96	0.56	0.69	0.85	1.04	1.27	1.55	1.90	2.33
0.24	0.96	0.53	0.65	0.80	0.99	1.22	1.50	1.84	2.27
0.26	0.97	0.49	0.61	0.76	0.94	1.16	1.44	1.76	2.21
0.28	0.97	0.46	0.56	0.72	0.90	1.11	1.38	1.72	2.15
0.30	0.97	0.43	0.54	0.68	0.85	1.06	1.33	1.66	2.08

Table 3. Odds Ratio

When an unexpected number of "bad" ships is found in period (t+1), the decision maker needs to calculate the $\mu_j(t+1)$ value reflecting the extreme odds ratio value. In Chapter II, it was shown that it is necessary to use a quite high variance to detect the changes in $\mu_j(t+1)$ caused by unexpected high or low $k_j(t+1)$ values. Having decided that $\mu_j(t+1)$ is different from the $\mu_j(t)$ obtained after completion of objective A, it is appropriate to disregard data from periods 1 through t, and base the estimate of $\mu_j(t+1)$ only on the inspection results found in period (t+1). Using the fraction of ships inspected that are found to be illegal we set:

$$\mu_j(t+1) = \frac{k_j(t)}{d_j(t)} \tag{3.3.a}$$

$$\sigma_j(t+1) = \varphi\mu_j(t+1) \tag{3.3.a}$$

where φ takes a value of 0.5, large enough to discriminate the possible changes in posterior inspections, based upon the ideas expressed in Section D, Chapter II. Then Equation (2.5) is used to estimate the asset allocation to each area according to the new μ_j's.

Having the updated μ_j and σ_j, the new α_j and β_j are calculated using Equations (2.3) and (2.4). At this point the decision maker restarts the inspection process following the μ_j estimation procedure under the Bayes procedure and using the conjugate beta distribution.

IV. MODEL IMPLEMENTATION.

A. INTRODUCTION

To test the reasonableness of the model, the decision cycle calculations are implemented using an Excel spreadsheet to observe the possible values obtained after an inspection is executed, the calculation of the posterior beta parameters and the corresponding inference for each X_j using the mean value of the posterior beta distribution, μ_j. This representation considers two objectives mentioned above: A) Reduce the uncertainty of X_j. and B) Maximize the expected number of illegal ships found in each inspection. Each objective is tested under four different scenarios. These scenarios represent inspection missions of length ℓ of one month. The assets inspection capacity is expressed by ships per day. Each scenario is defined by: a) Total number of available assets (N), b) True fraction of ships carrying illegal cargo, X_j, c) Accuracy level of the estimated mean fraction of ships carrying illegal cargo, φ value and d) Odds ratio ε value.

Two procedures are considered: 1) The decision maker, using the prior information or "intelligence", decides first to carry out several inspection missions to get a more accurate estimate of the fraction of ships with illegal cargo, μ_j, for each area j. Second, he/she allocates assets using the obtained μ_j to make inspections with the objective of maximizing the total number of ships detected with illegal cargo. 2) The decision maker decides not to minimize the uncertainty of the fraction of ships with illegal cargo, μ_j, as his initial objective. Instead, the planner chooses to maximize the total number of ships detected with illegal cargo, using only the prior information obtained by intelligence.

This model contains a single random factor in each period for each area: the number of illegal ships found in an area j during an inspection period, $K_j(t)$. Given this, a spreadsheet can be used as a simulation tool to demonstrate the analysis and the model plausibility.

B. EXAMPLE USING PROCEDURE 1.

Recall that in procedure 1, we start with intelligence estimates as the values of $\mu_j(0)$ in each area j and first reduce the uncertainty of these values. The model representation is displayed in Table 4 for one scenario. The number of inspected ships carrying illegal cargo are independent binomial random numbers with probability of 0.2. From intelligence reports the fraction of illegal ships in areas 1, 2 and 3 are 0.3, 0.4 and 0.25, respectively. Following the arguments at the end of Chapter II, we choose a coefficient of variation equal to 0.5. Using this value the standard deviations for areas 1, 2 and 3 are 0.15, 0.2 and 0.125, respectively. Other scenarios are tabulated in Appendix A.

OBJECTIVE: REDUCE UNCERTAINTY OF X_J

Total Assets(n)	25	φ Value	0.1
Real X_j (j=1,2,3)	0.2		

Variable description	Symbol	Area 1	Area 2	Area 3
Area information				
Ship arrival rate	λ_j	900	750	550
Assets inspection capacity	Cap_J	4	5	6
Initial mean fraction of illegal ships	$\mu_j(0)$	0.3	0.4	0.25
Initial S.D. fraction of illegal ships	$\sigma_j(0)$	0.150	0.200	0.125
Initial Beta parameters				
Alpha	$\alpha_j(0)$	2.50	2.00	2.75
Beta	$\beta_j(0)$	5.83	3.00	8.25
First Inspection				
Illegal ship rate	$\lambda_j\mu_j(0)$	270	300	137.5
Initial assets allocation	A_j1	11	10	4
# of ships to be inspected	q_j1	44	50	24
# of illegal ships found	$K_j(1)$	9	8	9
Updated Beta distribution				
Alpha	$\alpha_j(1)$	11.50	10.00	11.75
Beta	$\beta_j(1)$	40.83	45.00	23.25
Updated mean fraction of illegal ships	$\mu_j(1)$	0.22	0.18	0.34
Updated S.D. fraction of illegal ships	$\sigma_j(1)$	0.057	0.052	0.079
Coefficient of variation	CV	0.258	0.283	0.234
Updated illegal rate	$\lambda_j\mu_j(1)$	197.771	136	185

Table 4. Model Representation. Reduce the uncertainty of X_j

Second Inspection				
Assets allocation	A_j2	11	10	4
# of ships to be inspected	d_j2	44	50	24
# of illegal ships found	K_j2	10	7	5
Updated Beta Distribution				
Alpha	$\alpha_j(2)$	21.50	17.00	16.75
Beta	$\beta_j(2)$	74.83	88.00	42.25
Updated mean fraction of illegal ships	$\mu_j(2)$	0.22	0.16	0.28
Updated S.D. of fraction of illegal ships	$\sigma_j(2)$	0.042	0.036	0.058
Coefficient of variation	CV	0.189	0.221	0.205
Third Inspection				
New illegal rate	$\lambda_j\mu_j(2)$	198	120	154
Assets allocation	A_j3	11	10	4
# of ships to be inspected	d_j3	44	50	24
# of illegal ships found	$K_j(3)$	7	10	4
Updated Beta distribution				
Alpha	$\alpha_j(3)$	28.50	27.00	20.75
Beta	$\beta_j(3)$	111.83	128.00	62.25
Updated mean fraction of illegal ships	$\mu_j(3)$	0.2	0.17	0.25
Updated S.D.fraction of illegal ships	$\sigma_j(3)$	0.034	0.030	0.047
Coefficient of variation	CV	0.149	0.150	0.178
Fourth Inspection				
New illegal rate	$\lambda_j\mu_j(3)$	180	127.5	137.5
Assets allocation	A_j4	11	10	4
# of ships to be inspected	d_j4	44	50	24
# of illegal ships found	K_j4	10	9	6
Updated Beta Distribution				
Alpha	$\alpha_j(4)$	38.50	36.00	26.75
Beta	$\beta_j(4)$	145.83	169.00	80.25
Updated mean fraction of illegal ships	$\mu_j(4)$	0.21	0.18	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(4)$	0.030	0.027	0.042
Coefficient of variation	CV	0.143	0.151	0.167
Fifth Inspection				
New illegal rate	$\lambda_j\mu_j(4)$	189	135	137.5
Assets allocation	A_j5	11	10	4
# of ships to be inspected	d_j5	44	50	24
# of illegal ships found	K_j5	6	8	3
Updated Beta Distribution				
Alpha	$\alpha_j(5)$	44.50	44.00	29.75
Beta	$\beta_j(5)$	183.83	211.00	101.25
Updated mean fraction of illegal ships	$\mu_j(5)$	0.19	0.17	0.23
Updated S.D. fraction of Illegal ships	$\sigma_j(5)$	0.0262	0.0236	0.0365
Coefficient of variation	CV	0.134	0.137	0.161
Sixth Inspection				
New Illegal rate	$\lambda_j\mu_j(5)$	175.401	129.412	124.905
Assets allocation	A_j6	11	10	4
# of ships to be inspected	d_j6	44	50	24
# of Illegal ships found	K_j6	8	5	4
Updated Beta Distribution				
Alpha	$\alpha_j(6)$	52.50	49.00	33.75
Beta	$\beta_j(6)$	219.83	256.00	121.25
Updated mean fraction of illegal ships	$\mu_j(6)$	0.19	0.16	0.22
Updated S.D. fraction of illegal ships	$\sigma_j(6)$	0.024	0.021	0.033
Coefficient of variation	CV	0.124	0.131	0.152

Table 4. Model Representation. Reduce the uncertainty of X_j. (Continuation).

Seventh Inspection				
New illegal rate	$\lambda_j\mu_j(6)$	173.501	120.492	119.758
Assets allocation	A_j7	11	10	4
# of ships to be inspected	d_j7	44	50	24
# of illegal ships found	K_j7	9	10	7
Updated Beta Distribution				
Alpha	$\alpha_j(7)$	61.50	59.00	40.75
Beta	$\beta_j(7)$	254.83	296.00	138.25
Updated mean fraction of illegal ships	$\mu_j(7)$	0.19	0.17	0.23
Updated S.D. fraction of illegal ships	$\sigma_j(7)$	0.022	0.020	0.031
Coefficient of variation	CV	0.114	0.119	0.137
Eigth Inspection				
New illegal rate	$\lambda_j\mu_j(7)$	174.974	124.648	125.209
Assets allocation	A_j8	11	10	4
# of ships to be inspected	d_j8	44	50	24
# of illegal ships found	K_j8	7	14	3
Updated Beta Distribution				
Alpha	$\alpha_j(8)$	68.50	73.00	43.75
Beta	$\beta_j(8)$	291.83	332.00	159.25
Updated mean fraction of illegal ships	$\mu_j(8)$	0.19	0.18	0.22
Updated S.D. fraction of illegal ships	$\sigma_j(8)$	0.0206	0.0191	0.0288
Coefficient of variation	CV	0.109	0.106	0.134
Ninth Inspection				
New illegal rate	$\lambda_j\mu_j(8)$	171.092	135.185	118.534
Assets allocation	A_j9	11	10	4
# of ships to be inspected	d_j9	44	50	24
# of illegal ships found	K_j9	9	13	4
Updated Beta Distribution				
Alpha	$\alpha_j(9)$	77.50	86.00	47.75
Beta	$\beta_j(9)$	326.83	369.00	179.25
Updated mean fraction of illegal ships	$\mu_j(9)$	0.19	0.19	0.21
Updated S.D. fraction of illegal ships	$\sigma_j(9)$	0.020	0.018	0.027
Coefficient of variation	CV	0.096	0.092	0.121
Tenth Inspection				
New illegal rate	$\lambda_j\mu_j(9)$	172.506	141.758	115.694
Assets allocation	A_j10	11	10	4
# of ships to be inspected	d_j10	44	50	24
# of illegal ships found	K_j10	9	8	4
Updated Beta Distribution				
Alpha	$\alpha_j(10)$	86.50	94.00	51.75
Beta	$\beta_j(10)$	361.83	411.00	199.25
Updated mean fraction of illegal ships	$\mu_j(10)$	0.19	0.19	0.21
Updated S.D. fraction of illegal ships	$\sigma_j(10)$	0.019	0.017	0.025
Coefficient of variation	CV	0.096	0.093	0.124

Table 4. Model Representation. Reduce the uncertainty of X_j. (Continuation).

Eleventh Inspection				
New illegal rate	$\lambda_j\mu_j(10)$	173.643	139.604	113.396
Assets allocation	A_j11	11	10	4
# of ships to be inspected	d_j11	44	50	24
# of illegal ships found	K_j11	12	10	4
Updated Beta Distribution				
Alpha	$\alpha_j(11)$	98.50	104.00	55.75
Beta	$\beta_j(11)$	393.83	451.00	219.25
Updated mean fraction of illegal ships	$\mu_j(11)$	0.20	0.19	0.20
Updated S.D. fraction of illegal ships	$\sigma_j(11)$	0.018	0.017	0.024
Coefficient of variation	CV	0.090	0.088	0.101
Twelfth Inspection				
New illegal rate	$\lambda_j\mu_j(11)$	180.061	140.541	111.5
Assets allocation	A_j12	11	10	4
# of ships to be inspected	d_j12	44	50	24
# of illegal ships found	K_j12	6	6	4
Updated Beta Distribution				
Alpha	$\alpha_j(12)$	104.50	110.00	59.75
Beta	$\beta_j(12)$	431.83	495.00	239.25
Updated fraction of illegal ships	$\mu_j(12)$	0.19	0.18	0.20
Updated S.D. fraction of illegal ships	$\sigma_j(12)$	0.01709	0.01567	0.02309
Coefficient of variation	CV	0.088	0.086	0.101

Table 4. Model Representation. Reduce the uncertainty of X_j. (Continuation).

The following observations are made from the results in Table 4.

1. CV reduction as a measure of uncertainty of X_j.

Figure 7 illustrates the CV reduction for each area. Notice that for the area with bigger initial variance, area 3 , it is necessary to carry out more inspection missions to achieve a variance that satisfies $\sigma_j = 0.1\mu_j$. For this example, it took nine months in areas 1 and 2, and 12 months in area 3. This shows the importance of intelligence related to this type of operations. The number of time periods required to obtain a good μ_j estimation depends on how accurate the intelligence is.

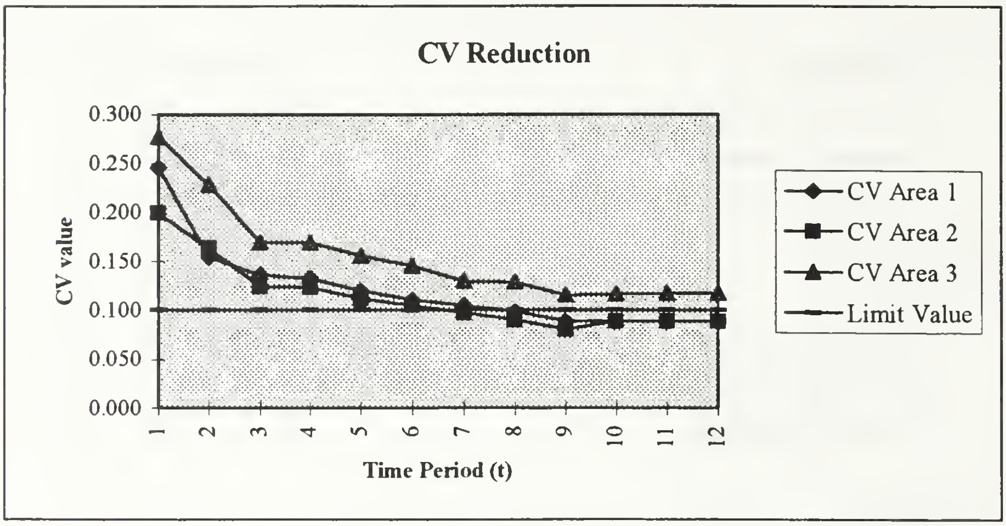

Figure 7. Coefficient of Variation Reduction

2. All the estimated fraction of ships carrying illegal cargo, $\mu_j(t)$ converge to the unknown real proportion , 0.2 in all areas.

Figure 8 displays how the $\mu_j(t)$'s values get closer to the real proportion of illegal ships in each area This result indicates that the model can be used as an adequate tool to get good approximations to the real proportion.

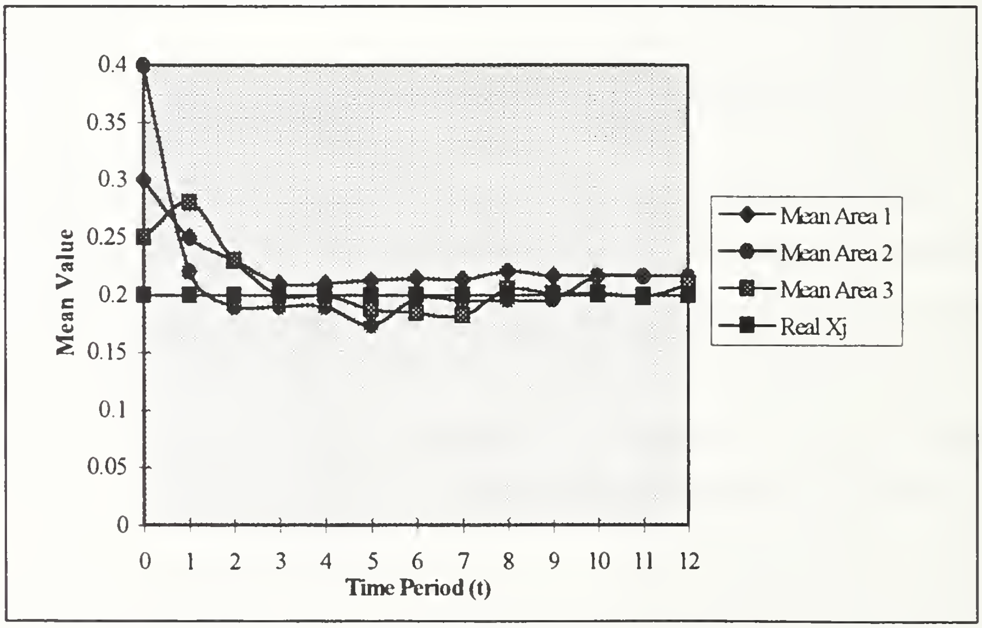

Figure 8. Estimated fraction of ships carrying illegal cargo, $\mu_{j,}$.

Having reduced the uncertainties on each μ_j to within a specified limit, assets are now allocated to maximize the number of illegal ships found.

Table 5 shows the process using this objective. Equation (2.5) is used to recalculate the assets allocated to each area j at the start of every period.

OBJECTIVE: MAXIMIZE NUMBER OF ILLEGAL SHIPS FOUND					
		95% ODDS RATIO BOUNDS	Area 1	Area 2	Area 3
		LOWER	0.36	0.43	0.44
Total Assets (n)	25	90	1.99	1.96	1.72
Real X_j (j=1,2,3)	0.2				
Variable description		**Symbol**	**Area 1**	**Area 2**	**Area 3**
Area information					
Ship arrival rate		λ_j	900	750	550
Assets inspection capacity		Cap_J	4	5	6
Initial mean fraction of illegal ships		$\mu_j(0)$	0.19	0.18	0.20
Initial S.D. fraction of illegal ships		$\sigma_j(0)$	0.019	0.018	0.020
Initial Beta parameters		$\alpha_j(0)$	80.81	81.82	79.80
Alpha		$\beta_j(0)$	344.51	372.74	319.20
Beta					
First Inspection					
Illegal ship rate		$\lambda_j\mu_j(0)$	171	135	110
Initial assets allocation		$A_j(1)$	12	8	5
# of ships to be inspected		$d_j(1)$	48	40	30
Expected number of illegal ships		$E[Kj(1)]$	9.6	8.0	6.0
# of illegal ships found		$K_j(1)$	7	7	5
Posterior Odds Ratio		$P(X_j>\mu_j(0))/P(Xj<\mu_j(0))$	0.65	0.93	0.79
Updated mean fraction of illegal ships		$\mu_j(1)$	0.19	0.18	0.20
Updated S.D. fraction of illegal ships		$\sigma_j(1)$	0.02	0.018	0.020
Updated Beta parameters					
Alpha		$\alpha_j(1)$	80.81	81.82	79.80
Beta		$\beta_j(1)$	344.51	372.74	319.20
Updated illegal rate		$\lambda_j\mu_j(1)$	171	135	110
Second Inspection					
New assets allocation		A_j2	12	8	5
# of ships to be inspected		d_j2	48	40	30
Expected number of illegal ships		$E[Kj(2)]$	9.6	8.0	6.0
# of illegal ships found		K_j2	9	8	5

Table 5. Maximizing the number of illegal ships found.

Posterior Odds Ratio	$P(X_j>\mu_j(1))/P(Xj<\mu_j(1))$	0.94	1.12	0.79
Updated mean fraction of illegal ships	$\mu_j(2)$	0.19	0.18	0.20
Updated S.D. fraction of illegal ships	$\sigma_j(2)$	0.019	0.018	0.019
Updated Beta parameters				
Alpha	$\alpha_j(2)$	80.81	81.82	79.80
Beta	$\beta_j(2)$	344.51	372.74	319.20
New illegal rate	$\lambda_j\mu_j(2)$	171	135	110
Third Inspection				
New assets allocation	A_j3	12	8	5
# of ships to be inspected	d_j3	48	40	30
Expected value of illegal ships	$E[Kj(3)]$	9.6	8.0	6.0
# of illegal ships found	$K_j(3)$	7	7	2
Posterior Odds Ratio	$P(X_j>\mu_j(2))/P(Xj<\mu_j(2))$	0.65	0.93	0.44
Updated mean fraction of illegal ships	$\mu_j(3)$	0.19	0.18	0.20
Updated S.D. fraction of illegal ships	$\sigma_j(3)$	0.019	0.018	0.020
Updated Beta parameters				
Alpha	$\alpha_j(3)$	80.81	81.82	79.80
Beta	$\beta_j(3)$	344.51	372.74	319.20
New illegal rate	$\lambda_j\mu_j(3)$	171	135	110
Fourth Inspection				
New assets allocation	$A_{j(}4)$	12	8	5
# of ships to be inspected	$d_{j(}4)$	48	40	30
Expected value of illegal ships	$E[Kj(4)]$	9.6	8.0	6.0
# of illegal ships found	$K_{j(}4)$	5	7	10
Posterior Odds Ratio	$P(X_j>\mu_j(3))/P(Xj<\mu_j(3))$	0.44	0.93	2.09
Updated mean fraction of illegal ships	$\mu_j(4)$	0.19	0.18	0.33
Updated S.D. fraction of illegal ships	$\sigma_j(4)$	0.02	0.02	0.17
Update Beta Distribution				
Alpha	$\alpha_j(4)$	80.81	81.82	2.33
Beta	$\beta_j(4)$	344.51	372.74	4.67
New illegal rate	$\lambda_j\mu_j(4)$	171	135	183
Fifth Inspection				
New assets allocation	$A_{j(}5)$	11	7	8
# of ships to be inspected	$d_{j(}5)$	44	35	48
Expected value of illegal ships	$E[Kj(5)]$	8.8	7.0	19.2
# of illegal ships found	$K_{j(}5)$	8	6	13
Posterior Odds Ratio	$P(X_j>\mu_j(4))/P(Xj<\mu_j(4))$	0.90	0.91	0.22
Updated mean fraction of illegal ships	$\mu_j(5)$	0.19	0.18	0.27
Updated S.D. fraction of illegal ships	$\sigma_j(5)$	0.02	0.02	0.14
Update Beta Distribution				
Alpha	$\alpha_j(5)$	80.81	81.82	2.65
Beta	$\beta_j(5)$	344.51	372.74	7.12
New illegal rate	$\lambda_j\mu_j(5)$	171	135	149

Table 5. Maximizing the number of illegal ships found. (Continuation)

The observations obtained from this simulated decision process are the following:

3. Observation 3. The odds ratio criterion

Table 6 presents the odds ratio value obtained for the example presented in Table 5. Except for area 3 during the fourth and fifth inspection, all the odds values are inside the ε values bounds,. From the results, it is clear that when the odds ratio value is out of bounds the model calculates the new assets allocation as a response of the change in the estimated fraction of ships carrying illegal cargo, μ_j. Even though a small change might be considered "normal" due to the process randomness, the model estimation is quite robust and calculates the appropriate asset allocation as result of the new μ_j's values. Any odds ratio value out of limits might be used as an indicator or warning for assets allocation in future inspection periods. In fact, the decision maker may select a desired X_j value and use the odds ratio as a "sensor" value to detect a change of unlawful events of this nature.

	POSTERIOR ODDS RATIO		
INSPECTION	**Area 1**	**Area 2**	**Area 3**
1ST	0.645	0.926	0.793
2ND	0.941	1.116	0.793
3RD	0.65	0.93	0.44
4TH	0.438	0.926	2.087
5TH	0.90	0.91	0.22

Table 6.Posterior Odds Ratio.

4. Observation 4: Maximum number of illegal ships found.

Table 7 displays the number of ships found during the 5 inspection periods for the example in Table 5. Note that the number of illegal ships found is close to the predicted expected number of illegal ships calculated according to the updated μ_j values. These results are quite satisfactory under the assumptions presented in this thesis. They demonstrate the feasibility of getting acceptable outcomes given the number of assets available.

EXPECTED NUMBER OF ILLEGAL SHIPS				
INSPECTION	Area 1	Area 2	Area 3	Total
1ST	7	7	5	19
2ND	9	8	5	22
3RD	10	8	6	19
4TH	10	8	6	24
5TH	9	7	19	35
				119

# ILLEGAL SHIPS FOUND				
INSPECTION	Area 1	Area 2	Area 3	Total
1ST	7	7	5	19
2ND	9	8	5	22
3RD	7	7	2	16
4TH	5	7	10	22
5TH	8	6	13	27
				106

Table 7.Expected Number of Illegal ships vs Number of Illegal ships found.

C. EXAMPLE USING PROCEDURE 2.

Sometimes the decision maker is constrained by time and other operational requirements. So, he/she decides to execute inspection missions to enforce laws trying to capture the maximum number of ships with illegal cargo starting with the intelligence of the μ_j's in each area. Recall from Table 4 that our initial estimates are 0.3, 0.4 and 0.25 for areas 1, 2 and 3 respectively. Table 8 presents the results of a simulated inspection process where the estimates of $\alpha_j(t)$, $\beta_j(t)$, $\mu_j(t)$ and $\sigma_j(t)$ are obtained using the Bayesian procedure. The assets are reallocated using Equation (2.5).

OBJECTIVE: MAXIMIZE NUMBER OF ILLEGAL SHIPS FOUND				
Total Assets (n)	25			
Real X_j	0.2			
Variable description	**Symbol**	**Area 1**	**Area 2**	**Area 3**
Area information				
Ship arrival rate	λ_j	900	750	550
Assets inspection capacity	Cap_J	4	5	6
Initial mean fraction of illegal ships	$\mu_j(0)$	0.30	0.40	0.25
Initial S.D. fraction of illegal ships	$\sigma_j(0)$	0.1500	0.2000	0.1250
Initial Beta parameters				
Alpha	$\alpha_j(0)$	2.50	2.00	2.75
Beta	$\beta_j(0)$	5.83	3.00	8.25
First Inspection				
Illegal ship rate	$\lambda_j\mu_j(0)$	270	300	138
Initial assets allocation	$A_j(1)$	11	10	4
# of ships to be inspected	$d_j(1)$	44	50	24
Expected number of illegal ships	$E[Kj(1)]$	8.8	10.0	4.8
# of illegal ships found	$K_j(1)$	9	12	5
Updated Beta parameters				
Alpha	$\alpha_j(1)$	11.50	14.00	7.75
Beta	$\beta_j(1)$	40.83	41.00	27.25
Updated mean fraction of illegal ships	$\mu_j(1)$	0.22	0.25	0.22
Updated S.D. fraction of illegal ships	$\sigma_j(1)$	0.06	0.08	0.07
Updated illegal rate	$\lambda_j\mu_j(1)$	197.77	190.91	121.79
Second Inspection				
New assets allocation	$A_{j(}2)$	11	9	5
# of ships to be inspected	$d_{j(}2)$	44	45	30
Expected number of illegal ships	$E[Kj(2)]$	8.8	9.0	6.0
# of illegal ships found	$K_{j(}2)$	9	12	6
Updated Beta parameters				
Alpha	$\alpha_j(2)$	20.50	26.00	13.75
Beta	$\beta_j(2)$	75.83	74.00	51.25
Updated mean fraction of illegal ships	$\mu_j(2)$	0.21	0.26	0.21
Updated S.D. fraction of illegal ships	$\sigma_j(2)$	0.04	0.04	0.05
New illegal rate	$\lambda_j\mu_j(2)$	191.52	195.00	116.35
Third Inspection				
New assets allocation	$A_{j(}3)$	11	9	5
# of ships to be inspected	$d_{j(}3)$	44	45	30
Expected value of illegal ships	$E[Kj(3)]$	8.8	9.0	6.0
# of illegal ships found	$K_j(3)$	10	14	5
Updated Beta parameters				
Alpha	$\alpha_j(3)$	30.50	40.00	18.75
Beta	$\beta_j(3)$	109.83	105.00	76.25
Updated mean fraction of illegal ships	$\mu_j(3)$	0.22	0.28	0.20
Updated S.D. fraction of illegal ships	$\sigma_j(3)$	0.03	0.04	0.04
New illegal rate	$\lambda_j\mu_j(3)$	195.61	206.90	108.55

Table 8. Maximizing the number of illegal ships found, using only "intelligence".

Fourth Inspection				
New assets allocation	$A_j(4)$	11	10	4
# of ships to be inspected	$d_j(4)$	44	50	24
Expected value of illegal ships	$E[Kj(4)]$	8.8	10.0	4.8
# of illegal ships found	$K_j(4)$	8	10	4
Update Beta Distribution				
Alpha	$\alpha_j(4)$	38.50	50.00	22.75
Beta	$\beta_j(4)$	145.83	145.00	98.25
Updated mean fraction of illegal ships	$\mu_j(4)$	0.21	0.28	0.19
Updated S.D. fraction of illegal ships	$\sigma_j(4)$	0.03	0.03	0.04
New illegal rate	$\lambda_j\mu_j(4)$	187.97	192.31	105.15
Fifth Inspection				
New assets allocation	$A_j(5)$	11	9	4
# of ships to be inspected	$d_j(5)$	44	45	24
Expected value of illegal ships	$E[Kj(5)]$	8.8	9.0	4.8
# of illegal ships found	$K_j(5)$	9	8	4
Update Beta Distribution				
Alpha	$\alpha_j(5)$	47.50	58.00	26.75
Beta	$\beta_j(5)$	180.83	184.00	118.25
Updated mean fraction of illegal ships	$\mu_j(5)$	0.21	0.23	0.19
Updated S.D. fraction of illegal ships	$\sigma_j(5)$	0.03	0.03	0.03
New illegal rate	$\lambda_j\mu_j(5)$	187.23	175.00	102.88

Table 8.(Continuation).

From this examples the remarks obtained are the following:

1. **With high uncertainty of μ_j., the odds criterion can not be used for asset allocation.**

Table 9 displays the asset allocation and the mean of the fraction of illegal ships carrying illegal cargo from the Bayesian procedure. The initial high variability or uncertainty of X_j produces a very wide confidence interval. Therefore, after inspection results, the obtained odds value does not offer clear information for asset allocation. If it is used would cause large changes in the asset allocation.

	ASSETS				MEAN		
INSPECTION	Area 1	Area 2	Area 3	INSPECTION	Area 1	Area 2	Area 3
1ST	11	10	4	1ST	0.239	0.236	0.136
2ND	13	9	3	2ND	0.168	0.220	0.146
3RD	11	10	4	3RD	0.165	0.260	0.166
4TH	10	11	4	4TH	0.183	0.234	0.146
5TH	11	10	4	5TH	0.192	0.220	0.150

Table 9.Assets Allocation.

Even though the odds value criterion is not appropiate for this situation, the decision maker can decide the asset allocation by the weighted distribution expressed by Equation (2.5). In this case, the algorithm offers a good alternative when the planner desires to get a satisfactory inspection result and he/she can not obtain a good estimate of the fraction of ships that are transporting illegal cargo. Table 10 shows a comparison of the expected number of illegal ships and the number of illegal ships found for the example in Table 8. The results are quite acceptable, in spite of the inaccurate initial estimation of μ_j.

EXPECTED NUMBER OF ILLEGAL SHIPS					# ILLEGAL SHIPS FOUND			
INSPECTION	Area 1	Area 2	Area 3	Total	Area 1	Area 2	Area 3	Total
1ST	9	12	5	26	9	12	5	26
2ND	9	12	6	27	9	12	6	27
3RD	9	9	6	26	10	14	5	29
4TH	9	10	5	24	8	10	4	22
5TH	9	9	5	23	9	6	4	19
				125				123

Table 10. Expected number of illegal ships and number of illegal ships found.

2. Inspection results guide to the real value of X_j

Figure 9 displays the estimated μ_j values for each of the three regions. Even though the planner decides to carry out inspections with a rough estimation of μ_j, he can obtain estimated μ_j values with tendency to the real value of X_j.

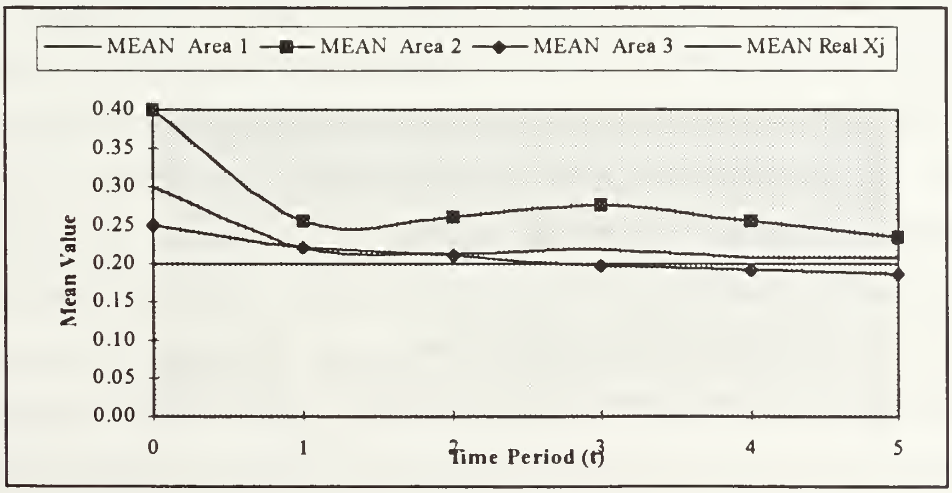

Figure 9.Estimated value of the fraction of ships with illegal cargo.

The asset allocation variability is low through the different inspection periods.

A comparison between the courses of action that a decision maker can execute using this model, drives the important observation that despite the decision maker uses Equation (2.5) under unreliable μ_j's values, the assets allocation keeps a regular pattern in each area during the time periods. Table 11 illustrates this point. After the first inspection, only the assets corresponding to area 2 present a significant change in the asset allocation. However, there exist an "equilibrium" after the inspections results update the estimate of the μ_j's values.

	ASSETS		
INSPECTION	Area 1	Area 2	Area 3
1ST	11.0	10.0	4.0
2ND	11.0	9.0	5.0
3RD	11.0	9.0	5.0
4TH	11.0	10.0	4.0
5TH	11.0	9.0	4.0

Table 11. Assets allocation.

3. Observation 4: Case 1 and Case 2 trade-offs.

In Case 2, unlike Case 1, the decision maker consider reallocation of assets between different assets of the end of every period t. In case 1, the asset allocation is fixed when the decision maker desires to get precise μ_j's values for each area. Then he/she reallocates assets to obtain the maximum number of ships carrying illegal cargo. Recalling that the underlying goal of this type of operations is to capture those ships involved in this kind of unlawful activity, it is useful to see what course of action offers the best tradeoff, considering time and the uncertainty involved in the value of the fraction of ships transporting illegal shipments, X_j. To analyze this point, Table 12 is used to illustrate the number of illegal ships found using both procedures. Note that the average number of illegal ships found per inspection period is slightly higher using procedure 2. From a practical point of view, the results show that using procedure 2 produces at least as good

results as procedure 1. 111 illegal ships were found in the first five periods using procedure 1, whereas this number was 123 using procedure 2.

	Number of Illegal ships found	Number of Time Periods	Number of Illegal ships found per period
PROCEDURE 1	373	17	21.94
PROCEDURE 2	123	5	24.60

Table 12. Number of Illegal ships found through Procedure 1 and Procedure 2.

In conclusion, it is reasonable to conclude that procedure 2 is better from a practical consideration. However, procedure 1 represents a useful method to gather important information or intelligence related with contraband or illegal shipment. It gives a clear insight about the presence of these activities. As it was expressed before, an accurate estimate of the fraction of ships transporting illegal cargo gives the decision maker a favorable starting point to ensure optimal results in the achievement of Control Maritime missions.

V. CONCLUSIONS AND FURTHER RESEARCH.

A. CONCLUSIONS

The model developed in this thesis provides an analytical procedure to determine the best allocation of assets for the execution of a Maritime Control mission in closed areas. Asset allocation is based on the fraction of ships that are carrying illegal cargo in each area j during a time period t. This model uses forecast information or "intelligence" to deploy such assets according to a Bayesian estimation of the illegal fraction. Few quantitative data related to this issue are available, which makes difficult the use of classical statistical methods.

The model estimates the proportion or fraction of ships carrying illegal cargo which heads to harbor through natural straits or channels. Using the available "intelligence" as input to generate an initial estimate of this fraction, the model defines a decision making process as a guide to the planner for asset allocation. The initial allocation decision is based upon a weighted distribution using the initial estimated mean of the fraction of illegal ships with illegal cargo, μ_j. Those areas to be inspected with higher fractions will receive more assets. The model demonstrates that if the decision maker decides to get a precise estimation of μ_j by carrying out several inspection with a fixed number of assets in each area, then he/she can maximize the number of illegal ships found with little change in the asset allocation. On the contrary, when the planner decides to make inspections only with the prior information from intelligence, he/she may reallocate assets more frequently, and more inspections are needed to get a desired precision for the estimated μ_j values. Even though this thesis keeps separate the logistics considerations related to the assets allocation, the fact remains that reducing the uncertainty of the μ_j value should be helpful in minimizing the complexity of the allocation problem.

This thesis involves intelligence as a quantitative input into the decision making process. The Bayesian approach makes possible the use of "subjective" information into the analytical model. The model is used to demonstrate that "intelligence" can be translated as an important estimator in this stochastic analysis.

In summary, this model represents a satisfactory starting point to obtain insights into the problem here presented. It presents an adequate alternative to improve the planning process of the type of Maritime Control missions presented.

B. FURTHER RESEARCH

Many of the model assumptions represent interesting areas for further investigations and potential model enrichment. For example:

1. Time to inspect a ship

The formulation of a stochastic model could provide an excellent tool to estimate this parameter. The associated randomness with it makes difficult the determination of the required number of assets for an inspection mission. Sea state, ship type to be inspected, assets capability, training, etc are some of the factors involved in this issue.

2. Probability of transporting illegal cargo

For this study, this value was assumed constant for every ship. However, it is known that some ships are more likely than others for transporting illegal cargo. Also, preceding data or frequency of ship type involved in this type of outlaw activities, may be used to develop a forecast model for determining this probability so that ships can be ranked based on this value.

3. Incorporation of Logistics restrictions

The scheduling or assets allocation implies constraints or considerations based on maintenance, personnel rotation, budgets and other conditions which influence ship distribution. Therefore, optimization models are potential tools to evaluate these factors and to give additional inputs to the planning of Maritime Control missions.

4. Illegal shipment detection

The design of a stochastic model or an optimization model might be possible to estimate the probability of detecting an illegal shipments on board. The measurement of this value represent an important enhancement to the decision making process related to Maritime Control planning.

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APPENDIX A. MODEL IMPLEMENTATION SCENARIOS

In this appendix the additional model implementation scenarios are collected to show how the model evaluates fictitious Maritime Control missions.

The scenarios are defined by the following factors:

a) Number of Assets availables, N.

b) Real fraction of illegal ships transporting illegal cargo in each area,X_j. However, in this example, it was assumed that this value is equal for all the areas.

c) Accuracy level of the estimated mean of ships carrying illegal cargo, μ_j.

d) Odds ratio confidence interval, ε value.

Also, after the completion of each mission, the results are tabulated to illustrate the inference of results and other considerations.

OBJECTIVE: REDUCE UNCERTAINTY OF X_J

Total Assets(n)	10	φ Value	0.1
Real X_j (j=1,2,3)	0.2		

Variable description	Symbol	Area 1	Area 2	Area 3
Area information				
Ship arrival rate	λ_j	900	750	550
Assets inspection capacity	Cap_J	4	5	6
Initial mean fraction of illegal ships	$\mu_j(0)$	0.3	0.4	0.25
Initial S.D. fraction of illegal ships	$\sigma_j(0)$	0.010	0.015	0.020
Initial Beta parameters				
Alpha	$\alpha_j(0)$	629.70	426.27	116.94
Beta	$\beta_j(0)$	1469.30	639.40	350.81
First Inspection				
Illegal ship rate	$\lambda_j\mu_j(0)$	270	300	137.5
Initial assets allocation	A_j1	4	4	2
# of ships to be inspected	d_j1	16	20	12
# of illegal ships found	$K_j(1)$	0	3	3
Updated Beta distribution				
Alpha	$\alpha_j(1)$	629.70	429.27	119.94
Beta	$\beta_j(1)$	1485.30	656.40	359.81
Updated mean fraction of illegal ships	$\mu_j(1)$	0.30	0.40	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(1)$	0.010	0.015	0.020
Coefficient of variation	CV	0.033	0.038	0.079
Updated illegal rate	$\lambda_j\mu_j(1)$	267.957	297	138
Second Inspection				
Assets allocation	A_j2	4	4	2
# of ships to be inspected	d_j2	16	20	12
# of illegal ships found	K_j2	6	3	3
Updated Beta Distribution				
Alpha	$\alpha_j(2)$	635.70	432.27	122.94
Beta	$\beta_j(2)$	1495.30	673.40	368.81
Updated mean fraction of illegal ships	$\mu_j(2)$	0.3	0.39	0.25
Updated S.D. of fraction of illegal ships	$\sigma_j(2)$	0.010	0.015	0.020
Coefficient of variation	CV	0.033	0.038	0.078
Third Inspection				
New illegal rate	$\lambda_j\mu_j(2)$	270	293	138
Assets allocation	A_j3	4	4	2
# of ships to be inspected	d_j3	16	20	12
# of illegal ships found	$K_j(3)$	3	5	2

Table 13.Procedure 1. Scenario 1

Updated Beta distribution				
Alpha	$\alpha_j(3)$	638.70	437.27	124.94
Beta	$\beta_j(3)$	1508.30	688.40	378.81
Updated mean fraction of illegal ships	$\mu_j(3)$	0.3	0.39	0.25
Updated S.D.fraction of illegal ships	$\sigma_j(3)$	0.010	0.015	0.019
Coefficient of variation	CV	0.039	0.047	0.088
Fourth Inspection				
New illegal rate	$\lambda_j\mu_j(3)$	270	292.5	137.5
Assets allocation	A_j4	4	4	2
# of ships to be inspected	d_j4	16	20	12
# of illegal ships found	K_j4	4	4	2
Updated Beta Distribution				
Alpha	$\alpha_j(4)$	642.70	441.27	126.94
Beta	$\beta_j(4)$	1520.30	704.40	388.81
Updated mean fraction of illegal ships	$\mu_j(4)$	0.3	0.39	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(4)$	0.010	0.014	0.019
Coefficient of variation	CV	0.033	0.037	0.077
Fifth Inspection				
New illegal rate	$\lambda_j\mu_j(4)$	270	292.5	137.5
Assets allocation	A_j5	4	4	2
# of ships to be inspected	d_j5	16	20	12
# of illegal ships found	K_j5	6	5	1
Updated Beta Distribution				
Alpha	$\alpha_j(5)$	648.70	446.27	127.94
Beta	$\beta_j(5)$	1530.30	719.40	399.81
Updated mean fraction of illegal ships	$\mu_j(5)$	0.30	0.38	0.24
Updated S.D. fraction of illegal ships	$\sigma_j(5)$	0.0098	0.0142	0.0186
Coefficient of variation	CV	0.033	0.037	0.077
Sixth Inspection				
New illegal rate	$\lambda_j\mu_j(5)$	267.935	287.132	133.331
Assets allocation	A_j6	4	4	2
# of ships to be inspected	d_j6	16	20	12
# of illegal ships found	K_j6	3	3	2
Updated Beta Distribution				
Alpha	$\alpha_j(6)$	651.70	449.27	129.94
Beta	$\beta_j(6)$	1543.30	736.40	409.81
Updated mean fraction of illegal ships	$\mu_j(6)$	0.30	0.38	0.24
Updated S.D. fraction of illegal ships	$\sigma_j(6)$	0.010	0.014	0.018
Coefficient of variation	CV	0.033	0.037	0.076

Table 13.Procedure 1. Scenario 1

Seventh Inspection				
New illegal rate	$\lambda_j\mu_j(6)$	267.212	284.186	132.405
Assets allocation	A_j7	4	4	2
# of ships to be inspected	d_j7	16	20	12
# of illegal ships found	K_j7	4	3	0
Updated Beta Distribution				
Alpha	$\alpha_j(7)$	655.70	452.27	129.94
Beta	$\beta_j(7)$	1555.30	753.40	421.81
Updated mean fraction of illegal ships	$\mu_j(7)$	0.30	0.38	0.24
Updated S.D. fraction of illegal ships	$\sigma_j(7)$	0.010	0.014	0.018
Coefficient of variation	CV	0.033	0.037	0.077
Eigth Inspection				
New illegal rate	$\lambda_j\mu_j(7)$	266.906	281.338	129.525
Assets allocation	A_j8	4	4	2
# of ships to be inspected	d_j8	16	20	12
# of illegal ships found	K_j8	4	4	4
Updated Beta Distribution				
Alpha	$\alpha_j(8)$	659.70	456.27	133.94
Beta	$\beta_j(8)$	1567.30	769.40	429.81
Updated mean fraction of illegal ships	$\mu_j(8)$	0.30	0.37	0.24
Updated S.D. fraction of illegal ships	$\sigma_j(8)$	0.0097	0.0138	0.0179
Coefficient of variation	CV	0.033	0.037	0.075
Ninth Inspection				
New illegal rate	$\lambda_j\mu_j(8)$	266.605	279.195	130.671
Assets allocation	A_j9	4	4	2
# of ships to be inspected	d_j9	16	20	12
# of illcgal ships found	K_j9	1	4	3
Updated Beta Distribution				
Alpha	$\alpha_j(9)$	660.70	460.27	136.94
Beta	$\beta_j(9)$	1582.30	785.40	438.81
Updatcd mean fraction of illegal ships	$\mu_j(9)$	0.29	0.37	0.24
Updated S.D. fraction of illcgal ships	$\sigma_j(9)$	0.010	0.014	0.018
Coefficient of variation	CV	0.033	0.037	0.074
Tenth Inspection				
New illcgal rate	$\lambda_j\mu_j(9)$	265.105	277.121	130.813
Assets allocation	A_j10	4	10	2
# of ships to be inspectcd	d_j10	16	50	12
# of illegal ships found	K_j10	1	7	0
Updated Beta Distribution				

Table 13.Procedure 1. Scenario 1

Alpha	$\alpha_j(10)$	661.70	467.27	136.94
Beta	$\beta_j(10)$	1597.30	828.40	450.81
Updated mean fraction of illegal ships	$\mu_j(10)$	0.29	0.36	0.23
Updated S.D. fraction of illegal ships	$\sigma_j(10)$	0.010	0.013	0.017
Coefficient of variation	CV	0.033	0.037	0.075
Eleventh Inspection				
New illegal rate	$\lambda_j\mu_j(10)$	263.625	270.479	128.142
Assets allocation	A_j11	4	10	2
# of ships to be inspected	d_j11	16	50	12
# of illegal ships found	K_j11	1	10	4
Updated Beta Distribution				
Alpha	$\alpha_j(11)$	662.70	477.27	140.94
Beta	$\beta_j(11)$	1612.30	868.40	458.81
Updated mean fraction of illegal ships	$\mu_j(11)$	0.29	0.35	0.23
Updated S.D. fraction of illegal ships	$\sigma_j(11)$	0.010	0.013	0.017
Coefficient of variation	CV	0.033	0.037	0.101
Twelfth Inspection				
New illegal rate	$\lambda_j\mu_j(11)$	262.167	266.002	129.247
Assets allocation	A_j12	4	10	2
# of ships to be inspected	d_j12	16	50	12
# of illegal ships found	K_j12	4	18	3
Updated Beta Distribution				
Alpha	$\alpha_j(12)$	666.70	495.27	143.94
Beta	$\beta_j(12)$	1624.30	900.40	467.81
Updated fraction of illegal ships	$\mu_j(12)$	0.29	0.35	0.24
Updated S.D. fraction of illegal ships	$\sigma_j(12)$	0.00949	0.0128	0.01714
Coefficient of variation	CV	0.033	0.036	0.101

Table 13.Procedure 1. Scenario 1

OBJECTIVE: MAXIMIZE NUMBER OF ILLEGAL SHIPS FOUND

			Area 1	Area 2	Area 3
		95% ODDS RATIO BOUNDS			
		LOWER	0.69	0.40	0.83
Total Assets (n)	10	85	4.98	2.60	3.52
Real X_j (j=1,2,3)	0.2				
Variable description		**Symbol**	**Area 1**	**Area 2**	**Area 3**
Area information					
Ship arrival rate		λ_j	900	750	550
Assets inspection capacity		Cap_J	4	5	6
Initial mean fraction of illegal ships		$\mu_j(0)$	0.28	0.34	0.23
Initial S.D. fraction of illegal ships		$\sigma_j(0)$	0.028	0.034	0.023
Initial Beta parameters		$\alpha_j(0)$	71.72	65.66	76.77
Alpha		$\beta_j(0)$	184.42	127.46	257.01
Beta					
First Inspection					
Illegal ship rate		$\lambda_j\mu_j(0)$	252	255	127
Initial assets allocation		$A_j(1)$	5	4	2
# of ships to be inspected		$d_j(1)$	20	20	12
Expected number of illegal ships		$E[Kj(1)]$	4.0	4.0	2.4
# of illegal ships found		$K_j(1)$	7	7	5
Posterior Odds Ratio		$P(X_j>\mu_j(0))/P(Xj<\mu_j(0))$	1.31	1.02	1.52
Updated mean fraction of illegal ships		$\mu_j(1)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships		$\sigma_j(1)$	0.03	0.034	0.023
Updated Beta parameters					
Alpha		$\alpha_j(1)$	71.72	65.66	76.77
Beta		$\beta_j(1)$	184.42	127.46	257.01
Updated illegal rate		$\lambda_j\mu_j(1)$	252	255	127
Second Inspection					
New assets allocation		A_j2	5	4	2
# of ships to be inspected		d_j2	20	20	12
Expected number of illegal ships		$E[Kj(2)]$	4.0	4.0	2.4
# of illegal ships found		K_j2	9	8	5
Posterior Odds Ratio		$P(X_j>\mu_j(1))/P(Xj<\mu_j(1))$	2.01	1.29	1.52
Updated mean fraction of illegal ships		$\mu_j(2)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships		$\sigma_j(2)$	0.028	0.034	0.019
Updated Beta parameters					
Alpha		$\alpha_j(2)$	71.72	65.66	76.77
Beta		$\beta_j(2)$	184.42	127.46	257.01
New illegal rate		$\lambda_j\mu_j(2)$	252	255	127

Table 13.Procedure 1. Scenario 1

Third Inspection				
New assets allocation	A_j3	5	4	2
# of ships to be inspected	d_j3	20	20	12
Expected value of illegal ships	$E[K_j(3)]$	4.0	4.0	2.4
# of illegal ships found	$K_j(3)$	7	7	2
Posterior Odds Ratio	$P(X_j>\mu_j(2))/P(Xj<\mu_j(2))$	1.31	1.02	0.83
Updated mean fraction of illegal ships	$\mu_j(3)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships	$\sigma_j(3)$	0.028	0.034	0.023
Updated Beta parameters				
Alpha	$\alpha_j(3)$	71.72	65.66	76.77
Beta	$\beta_j(3)$	184.42	127.46	257.01
New illegal rate	$\lambda_j\mu_j(3)$	252	255	127
Fourth Inspection				
New assets allocation	$A_j(4)$	5	4	2
# of ships to be inspected	$d_j(4)$	20	20	12
Expected value of illegal ships	$E[K_j(4)]$	4.0	4.0	2.4
# of illegal ships found	$K_j(4)$	5	7	10
Posterior Odds Ratio	$P(X_j>\mu_j(3))/P(Xj<\mu_j(3))$	0.85	1.02	4.39
Updated mean fraction of illegal ships	$\mu_j(4)$	0.28	0.34	0.83
Updated S.D. fraction of illegal ships	$\sigma_j(4)$	0.03	0.03	0.42
Update Beta Distribution				
Alpha	$\alpha_j(4)$	71.72	65.66	76.77
Beta	$\beta_j(4)$	184.42	127.46	257.01
New illegal rate	$\lambda_j\mu_j(4)$	252	255	458
Fifth Inspection				
New assets allocation	$A_j(5)$	3	3	4
# of ships to be inspected	$d_j(5)$	12	15	24
Expected value of illegal ships	$E[K_j(5)]$	2.4	3.0	9.6
# of illegal ships found	$K_j(5)$	3	4	13
Posterior Odds Ratio	$P(X_j>\mu_j(4))/P(Xj<\mu_j(4))$	0.90	0.75	0.00
Updated mean fraction of illegal ships	$\mu_j(5)$	0.28	0.34	0.54
Updated S.D. fraction of illegal ships	$\sigma_j(5)$	0.03	0.03	0.27
Update Beta Distribution				
Alpha	$\alpha_j(5)$	71.72	65.66	1.29
Beta	$\beta_j(5)$	184.42	127.46	1.09
New illegal rate	$\lambda_j\mu_j(5)$	252	255	298

Table 13.Procedure 1. Scenario 1

CV REDUCTION

t	CV Area 1	CV Area 2	CV Area 3
1	0.033	0.038	0.079
2	0.033	0.038	0.078
3	0.039	0.047	0.088
4	0.033	0.037	0.077
5	0.033	0.037	0.077
6	0.033	0.037	0.076
7	0.033	0.037	0.077
8	0.033	0.037	0.075
9	0.033	0.037	0.074
10	0.033	0.037	0.075
11	0.033	0.037	0.101
12	0.033	0.036	0.101

MEAN

t	Mean Area 1	Mean Area 2	Mean Area 3
0	0.3	0.4	0.25
1	0.30	0.40	0.25
2	0.3	0.39	0.25
3	0.3	0.39	0.25
4	0.3	0.39	0.25
5	0.30	0.38	0.24
6	0.30	0.38	0.24
7	0.30	0.38	0.24
8	0.30	0.37	0.24
9	0.29	0.37	0.24
10	0.29	0.36	0.23
11	0.29	0.35	0.23
12	0.29	0.29	0.24

Table 13.Procedure 1. Scenario 1

INSPECTIONS MISSIONS RESULTS   SCENARIO: 1

ASSETS

INSPECTION	Area 1	Area 2	Area 3
1ST	5.0	4.0	2.0
2ND	5.0	4.0	2.0
3RD	5.0	4.0	2.0
4TH	5.0	4.0	2.0
5TH	3.0	3.0	4.0

POSTERIOR ODDS RATIO

INSPECTION	Area 1	Area 2	Area 3
1ST	1.308	1.022	1.523
2ND	2.014	1.288	1.523
3RD	1.31	1.02	0.83
4TH	0.852	1.022	4.389
5TH	0.90	0.75	0.00

MEAN

INSPECTION	Area 1	Area 2	Area 3
1ST	0.280	0.340	0.230
2ND	0.280	0.340	0.230
3RD	0.280	0.340	0.230
4TH	0.280	0.340	0.833
5TH	0.280	0.340	0.542

EXPECTED NUMBER OF ILLEGAL SHIPS

INSPECTION	Area 1	Area 2	Area 3	Total
1ST	7	7	5	19
2ND	9	8	5	22
3RD	4	4	2	19
4TH	4	4	2	10
5TH	2	3	10	15
				85

ILLEGAL SHIPS FOUND

INSPECTION	Area 1	Area 2	Area 3	Total
1ST	7	7	5	19
2ND	9	8	5	22
3RD	7	7	2	16
4TH	5	7	10	22
5TH	3	4	13	20
				99

Table 13.Procedure 1. Scenario 1

OBJECTIVE: REDUCE UNCERTAINTY OF X_J

Total Assets(n)	15	φ Value	0.1
Real X_j (j=1,2,3)	0.2		

Variable description	Symbol	Area 1	Area 2	Area 3
Area information				
Ship arrival rate	λ_j	900	750	550
Assets inspection capacity	Cap_J	4	5	6
Initial mean fraction of illegal ships	$\mu_j(0)$	0.3	0.4	0.25
Initial S.D. fraction of illegal ships	$\sigma_j(0)$	0.010	0.015	0.020
Initial Beta parameters				
Alpha	$\alpha_j(0)$	629.70	426.27	116.94
Beta	$\beta_j(0)$	1469.30	639.40	350.81
First Inspection				
Illegal ship rate	$\lambda_j\mu_j(0)$	270	300	137.5
Initial assets allocation	A_j1	7	6	2
# of ships to be inspected	d_j1	28	30	12
# of illegal ships found	$K_j(1)$	4	12	2
Updated Beta distribution				
Alpha	$\alpha_j(1)$	633.70	438.27	118.94
Beta	$\beta_j(1)$	1493.30	657.40	360.81
Updated mean fraction of illegal ships	$\mu_j(1)$	0.30	0.40	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(1)$	0.010	0.015	0.020
Coefficient of variation	CV	0.033	0.037	0.079
Updated illegal rate	$\lambda_j\mu_j(1)$	268.138	300	136
Second Inspection				
Assets allocation	A_j2	7	6	2
# of ships to be inspected	d_j2	28	30	12
# of illegal ships found	K_j2	5	6	3
Updated Beta Distribution				
Alpha	$\alpha_j(2)$	638.70	444.27	121.94
Beta	$\beta_j(2)$	1516.30	681.40	369.81
Updated mean fraction of illegal ships	$\mu_j(2)$	0.3	0.39	0.25
Updated S.D. of fraction of illegal ships	$\sigma_j(2)$	0.010	0.015	0.019
Coefficient of variation	CV	0.033	0.037	0.078
Third Inspection				
New illegal rate	$\lambda_j\mu_j(2)$	270	293	138
Assets allocation	A_j3	7	6	2
# of ships to be inspected	d_j3	28	30	12
# of illegal ships found	$K_j(3)$	4	6	4

Table 13.Procedure 1. Scenario 2

Updated Beta distribution				
Alpha	$\alpha_j(3)$	642.70	450.27	125.94
Beta	$\beta_j(3)$	1540.30	705.40	377.81
Updated mean fraction of illegal ships	$\mu_j(3)$	0.29	0.39	0.25
Updated S.D.fraction of illegal ships	$\sigma_j(3)$	0.010	0.014	0.019
Coefficient of variation	CV	0.039	0.046	0.088
Fourth Inspection				
New illegal rate	$\lambda_j\mu_j(3)$	261	292.5	137.5
Assets allocation	A_j4	7	6	2
# of ships to be inspected	d_j4	28	30	12
# of illegal ships found	K_j4	4	4	3
Updated Beta Distribution				
Alpha	$\alpha_j(4)$	646.70	454.27	128.94
Beta	$\beta_j(4)$	1564.30	731.40	386.81
Updated mean fraction of illegal ships	$\mu_j(4)$	0.29	0.38	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(4)$	0.010	0.014	0.019
Coefficient of variation	CV	0.033	0.037	0.076
Fifth Inspection				
New illegal rate	$\lambda_j\mu_j(4)$	261	285	137.5
Assets allocation	A_j5	7	6	2
# of ships to be inspected	d_j5	28	30	12
# of illegal ships found	K_j5	4	6	3
Updated Beta Distribution				
Alpha	$\alpha_j(5)$	650.70	460.27	131.94
Beta	$\beta_j(5)$	1588.30	755.40	395.81
Updated mean fraction of illegal ships	$\mu_j(5)$	0.29	0.38	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(5)$	0.0096	0.0139	0.0188
Coefficient of variation	CV	0.033	0.037	0.075
Sixth Inspection				
New illegal rate	$\lambda_j\mu_j(5)$	261.559	283.959	137.5
Assets allocation	A_j6	7	6	2
# of ships to be inspected	d_j6	28	30	12
# of illegal ships found	K_j6	4	3	2
Updated Beta Distribution				
Alpha	$\alpha_j(6)$	654.70	463.27	133.94
Beta	$\beta_j(6)$	1612.30	782.40	405.81
Updated mean fraction of illegal ships	$\mu_j(6)$	0.29	0.37	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(6)$	0.010	0.014	0.019
Coefficient of variation	CV	0.033	0.037	0.075

Table 13.Procedure 1. Scenario 2

Seventh Inspection				
New illegal rate	$\lambda_j\mu_j(6)$	259.916	278.927	136.481
Assets allocation	A_j7	7	6	2
# of ships to be inspected	d_j7	28	30	12
# of illegal ships found	K_j7	5	9	2
Updated Beta Distribution				
Alpha	$\alpha_j(7)$	659.70	472.27	135.94
Beta	$\beta_j(7)$	1635.30	803.40	415.81
Updated mean fraction of illegal ships	$\mu_j(7)$	0.29	0.37	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(7)$	0.009	0.014	0.018
Coefficient of variation	CV	0.033	0.037	0.074
Eigth Inspection				
New illegal rate	$\lambda_j\mu_j(7)$	258.706	277.659	135.506
Assets allocation	A_j8	7	6	2
# of ships to be inspected	d_j8	28	30	12
# of illegal ships found	K_j8	6	10	1
Updated Beta Distribution				
Alpha	$\alpha_j(8)$	665.70	482.27	136.94
Beta	$\beta_j(8)$	1657.30	823.40	426.81
Updated mean fraction of illegal ships	$\mu_j(8)$	0.29	0.37	0.24
Updated S.D. fraction of illegal ships	$\sigma_j(8)$	0.0094	0.0134	0.0180
Coefficient of variation	CV	0.033	0.036	0.074
Ninth Inspection				
New illegal rate	$\lambda_j\mu_j(8)$	257.912	277.023	133.598
Assets allocation	A_j9	7	6	2
# of ships to be inspected	d_j9	28	30	12
# of illcgal ships found	K_j9	4	1	3
Updated Beta Distribution				
Alpha	$\alpha_j(9)$	669.70	483.27	139.94
Beta	$\beta_j(9)$	1681.30	852.40	435.81
Updated mean fraction of illegal ships	$\mu_j(9)$	0.28	0.36	0.24
Updated S.D. fraction of illegal ships	$\sigma_j(9)$	0.009	0.013	0.018
Coefficient of variation	CV	0.032	0.036	0.073
Tenth Inspection				
New illegal rate	$\lambda_j\mu_j(9)$	256.372	271.363	133.679
Assets allocation	A_j10	7	10	2
# of ships to be inspected	d_j10	28	50	12
# of illegal ships found	K_j10	8	8	1
Updated Beta Distribution				

Table 13.Procedure 1. Scenario 2

Alpha	$\alpha_j(10)$	677.70	491.27	140.94
Beta	$\beta_j(10)$	1701.30	894.40	446.81
Updated mean fraction of illegal ships	$\mu_j(10)$	0.28	0.35	0.24
Updated S.D. fraction of illegal ships	$\sigma_j(10)$	0.009	0.013	0.018
Coefficient of variation	CV	0.032	0.036	0.073
Eleventh Inspection				
New illegal rate	$\lambda_j\mu_j(10)$	256.381	265.901	131.885
Assets allocation	A_j11	7	10	2
# of ships to be inspected	d_j11	28	50	12
# of illegal ships found	K_j11	5	8	1
Updated Beta Distribution				
Alpha	$\alpha_j(11)$	682.70	499.27	141.94
Beta	$\beta_j(11)$	1724.30	936.40	457.81
Updated mean fraction of illegal ships	$\mu_j(11)$	0.28	0.35	0.24
Updated S.D. fraction of illegal ships	$\sigma_j(11)$	0.009	0.013	0.017
Coefficient of variation	CV	0.032	0.036	0.101
Twelfth Inspection				
New illegal rate	$\lambda_j\mu_j(11)$	255.268	260.82	130.164
Assets allocation	A_j12	7	10	2
# of ships to be inspected	d_j12	28	50	12
# of illegal ships found	K_j12	6	10	1
Updated Beta Distribution				
Alpha	$\alpha_j(12)$	688.70	509.27	142.94
Beta	$\beta_j(12)$	1746.30	976.40	468.81
Updated fraction of illegal ships	$\mu_j(12)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships	$\sigma_j(12)$	0.00913	0.01231	0.01709
Coefficient of variation	CV	0.032	0.036	0.101

Table 13.Procedure 1. Scenario 2

OBJECTIVE: MAXIMIZE NUMBER OF ILLEGAL SHIPS FOUND

		95% ODDS RATIO BOUNDS	Area 1	Area 2	Area 3
		LOWER	0.42	0.17	0.83
Total Assets (n)	15	85	2.92	1.17	3.52
Real X_j (j=1,2,3)	0.2				
Variable description		**Symbol**	**Area 1**	**Area 2**	**Area 3**
Area information					
Ship arrival rate		λ_j	900	750	550
Assets inspection capacity		Cap_J	4	5	6
Initial mean fraction of illegal ships		$\mu_j(0)$	0.28	0.34	0.23
Initial S.D. fraction of illegal ships		$\sigma_j(0)$	0.028	0.034	0.023
Initial Beta parameters		$\alpha_j(0)$	71.72	65.66	76.77
Alpha		$\beta_j(0)$	184.42	127.46	257.01
Beta					
First Inspection					
Illegal ship rate		$\lambda_j\mu_j(0)$	252	255	127
Initial assets allocation		$A_j(1)$	7	6	2
# of ships to be inspected		$d_j(1)$	28	30	12
Expected number of illegal ships		E[Kj(1)]	5.6	6.0	2.4
# of illegal ships found		$K_j(1)$	7	7	5
Posterior Odds Ratio		$P(X_j>\mu_j(0))/P(Xj<\mu_j(0))$	0.81	0.47	1.52
Updated mean fraction of illegal ships		$\mu_j(1)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships		$\sigma_j(1)$	0.03	0.034	0.023
Updated Beta parameters					
Alpha		$\alpha_j(1)$	71.72	65.66	76.77
Beta		$\beta_j(1)$	184.42	127.46	257.01
Updated illegal rate		$\lambda_j\mu_j(1)$	252	255	127
Second Inspection					
New assets allocation		A_j2	7	6	2
# of ships to be inspected		d_j2	28	30	12
Expected number of illegal ships		E[Kj(2)]	5.6	6.0	2.4
# of illegal ships found		K_j2	9	8	5
Posterior Odds Ratio		$P(X_j>\mu_j(1))/P(Xj<\mu_j(1))$	1.24	0.59	1.52
Updated mean fraction of illegal ships		$\mu_j(2)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships		$\sigma_j(2)$	0.028	0.034	0.019
Updated Beta parameters					
Alpha		$\alpha_j(2)$	71.72	65.66	76.77
Beta		$\beta_j(2)$	184.42	127.46	257.01
New illegal rate		$\lambda_j\mu_j(2)$	252	255	127

Table 13.Procedure 1. Scenario 2

Third Inspection				
New assets allocation	A_j3	7	6	2
# of ships to be inspected	d_j3	28	30	12
Expected value of illegal ships	$E[Kj(3)]$	5.6	6.0	2.4
# of illegal ships found	$K_j(3)$	7	7	2
Posterior Odds Ratio	$P(X_j>\mu_j(2))/P(Xj<\mu_j(2))$	0.81	0.47	0.83
Updated mean fraction of illegal ships	$\mu_j(3)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships	$\sigma_j(3)$	0.028	0.034	0.023
Updated Beta parameters				
Alpha	$\alpha_j(3)$	71.72	65.66	76.77
Beta	$\beta_j(3)$	184.42	127.46	257.01
New illegal rate	$\lambda_j\mu_j(3)$	252	255	127
Fourth Inspection				
New assets allocation	$A_{j(}4)$	7	6	2
# of ships to be inspected	$d_{j(}4)$	28	30	12
Expected value of illegal ships	$E[Kj(4)]$	5.6	6.0	2.4
# of illegal ships found	$K_{j(}4)$	5	7	10
Posterior Odds Ratio	$P(X_j>\mu_j(3))/P(Xj<\mu_j(3))$	0.53	0.47	4.39
Updated mean fraction of illegal ships	$\mu_j(4)$	0.28	0.34	0.83
Updated S.D. fraction of illegal ships	$\sigma_j(4)$	0.03	0.03	0.42
Update Beta Distribution				
Alpha	$\alpha_j(4)$	71.72	65.66	-0.17
Beta	$\beta_j(4)$	184.42	127.46	-0.03
New illegal rate	$\lambda_j\mu_j(4)$	252	255	458
Fifth Inspection				
New assets allocation	$A_{j(}5)$	5	4	6
# of ships to be inspected	$d_{j(}5)$	20	20	36
Expected value of illegal ships	$E[Kj(5)]$	4.0	4.0	14.4
# of illegal ships found	$K_{j(}5)$	4	2	13
Posterior Odds Ratio	$P(X_j>\mu_j(4))/P(Xj<\mu_j(4))$	0.69	0.31	0.00
Updated mean fraction of illegal ships	$\mu_j(5)$	0.28	0.34	0.36
Updated S.D. fraction of illegal ships	$\sigma_j(5)$	0.03	0.03	0.18
Update Beta Distribution				
Alpha	$\alpha_j(5)$	71.72	65.66	2.19
Beta	$\beta_j(5)$	184.42	127.46	3.88
New illegal rate	$\lambda_j\mu_j(5)$	252	255	199

Table 13.Procedure 1. Scenario 2

CV REDUCTION

t	CV Area 1	CV Area 2	CV Area 3
1	0.033	0.037	0.079
2	0.033	0.037	0.078
3	0.039	0.046	0.088
4	0.033	0.037	0.076
5	0.033	0.037	0.075
6	0.033	0.037	0.075
7	0.033	0.037	0.074
8	0.033	0.036	0.074
9	0.032	0.036	0.073
10	0.032	0.036	0.073
11	0.032	0.036	0.101
12	0.032	0.036	0.101

MEAN

t	Mean Area 1	Mean Area 2	Mean Area 3
0	0.3	0.4	0.25
1	0.30	0.40	0.25
2	0.3	0.39	0.25
3	0.29	0.39	0.25
4	0.29	0.38	0.25
5	0.29	0.38	0.25
6	0.29	0.37	0.25
7	0.29	0.37	0.25
8	0.29	0.37	0.24
9	0.28	0.36	0.24
10	0.28	0.35	0.24
11	0.28	0.35	0.24
12	0.28	0.28	0.23

Table 13.Procedure 1. Scenario 2

INSPECTIONS MISSIONS RESULTS SCENARIO: 2

ASSETS

INSPECTION	Area 1	Area 2	Area 3
1ST	7.0	6.0	2.0
2ND	7.0	6.0	2.0
3RD	7.0	6.0	2.0
4TH	7.0	6.0	2.0
5TH	5.0	4.0	6.0

POSTERIOR ODDS RATIO

INSPECTION	Area 1	Area 2	Area 3
1ST	0.812	0.469	1.523
2ND	1.239	0.592	1.523
3RD	0.81	0.47	0.83
4TH	0.529	0.469	4.389
5TH	0.69	0.31	0.00

MEAN

INSPECTION	Area 1	Area 2	Area 3
1ST	0.280	0.340	0.230
2ND	0.280	0.340	0.230
3RD	0.280	0.340	0.230
4TH	0.280	0.340	0.833
5TH	0.280	0.340	0.361

EXPECTED NUMBER OF ILLEGAL SHIPS

INSPECTION	Area 1	Area 2	Area 3	Total
1ST	7	7	5	19
2ND	9	8	5	22
3RD	6	6	2	19
4TH	6	6	2	14
5TH	4	4	14	22
				96

ILLEGAL SHIPS FOUND

INSPECTION	Area 1	Area 2	Area 3	Total
1ST	7	7	5	19
2ND	9	8	5	22
3RD	7	7	2	16
4TH	5	7	10	22
5TH	4	2	13	19
				98

Table 13.Procedure 1. Scenario 2

OBJECTIVE: REDUCE UNCERTAINTY OF X_J

Total Assets(n)	20	φ Value	0.1
Real X_j (j=1,2,3)	0.2		

Variable description	Symbol	Area 1	Area 2	Area 3
Area information				
Ship arrival rate	λ_j	900	750	550
Assets inspection capacity	Cap_J	4	5	6
Initial mean fraction of illegal ships	$\mu_j(0)$	0.3	0.4	0.25
Initial S.D. fraction of illegal ships	$\sigma_j(0)$	0.010	0.015	0.020
Initial Beta parameters				
Alpha	$\alpha_j(0)$	629.70	426.27	116.94
Beta	$\beta_j(0)$	1469.30	639.40	350.81
First Inspection				
Illegal ship rate	$\lambda_j\mu_j(0)$	270	300	137.5
Initial assets allocation	A_j1	9	8	3
# of ships to be inspected	d_j1	36	40	18
# of illegal ships found	$K_j(1)$	3	8	8
Updated Beta distribution				
Alpha	$\alpha_j(1)$	632.70	434.27	124.94
Beta	$\beta_j(1)$	1502.30	671.40	360.81
Updated mean fraction of illegal ships	$\mu_j(1)$	0.30	0.39	0.26
Updated S.D. fraction of illegal ships	$\sigma_j(1)$	0.010	0.015	0.020
Coefficient of variation	CV	0.033	0.037	0.077
Updated illegal rate	$\lambda_j\mu_j(1)$	266.712	295	141
Second Inspection				
Assets allocation	A_j2	9	8	3
# of ships to be inspected	d_j2	36	40	18
# of illegal ships found	K_j2	7	5	3
Updated Beta Distribution				
Alpha	$\alpha_j(2)$	639.70	439.27	127.94
Beta	$\beta_j(2)$	1531.30	706.40	375.81
Updated mean fraction of illegal ships	$\mu_j(2)$	0.29	0.38	0.25
Updated S.D. of fraction of illegal ships	$\sigma_j(2)$	0.010	0.014	0.019
Coefficient of variation	CV	0.033	0.037	0.076
Third Inspection				
New illegal rate	$\lambda_j\mu_j(2)$	261	285	138
Assets allocation	A_j3	9	8	3
# of ships to bc inspected	d_j3	36	40	18
# of illegal ships found	$K_j(3)$	6	13	1

Table 13.Procedure 1. Scenario 3

Updated Beta distribution				
Alpha	$\alpha_j(3)$	645.70	452.27	128.94
Beta	$\beta_j(3)$	1561.30	733.40	392.81
Updated mean fraction of illegal ships	$\mu_j(3)$	0.29	0.38	0.25
Updated S.D.fraction of illegal ships	$\sigma_j(3)$	0.010	0.014	0.019
Coefficient of variation	CV	0.039	0.046	0.086
Fourth Inspection				
New illegal rate	$\lambda_j\mu_j(3)$	261	285	137.5
Assets allocation	A_j4	9	8	3
# of ships to be inspected	d_j4	36	40	18
# of illegal ships found	K_j4	3	6	4
Updated Beta Distribution				
Alpha	$\alpha_j(4)$	648.70	458.27	132.94
Beta	$\beta_j(4)$	1594.30	767.40	406.81
Updated mean fraction of illegal ships	$\mu_j(4)$	0.29	0.37	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(4)$	0.010	0.014	0.019
Coefficient of variation	CV	0.033	0.037	0.075
Fifth Inspection				
New illegal rate	$\lambda_j\mu_j(4)$	261	277.5	137.5
Assets allocation	A_j5	9	8	3
# of ships to be inspected	d_j5	36	40	18
# of illegal ships found	K_j5	10	4	7
Updated Beta Distribution				
Alpha	$\alpha_j(5)$	658.70	462.27	139.94
Beta	$\beta_j(5)$	1620.30	803.40	417.81
Updated mean fraction of illegal ships	$\mu_j(5)$	0.29	0.37	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(5)$	0.0095	0.0135	0.0183
Coefficient of variation	CV	0.033	0.037	0.073
Sixth Inspection				
New illegal rate	$\lambda_j\mu_j(5)$	260.127	273.927	137.993
Assets allocation	A_j6	9	8	3
# of ships to be inspected	d_j6	36	40	18
# of illegal ships found	K_j6	7	10	4
Updated Beta Distribution				
Alpha	$\alpha_j(6)$	665.70	472.27	143.94
Beta	$\beta_j(6)$	1649.30	833.40	431.81
Updated mean fraction of illegal ships	$\mu_j(6)$	0.29	0.36	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(6)$	0.009	0.013	0.018
Coefficient of variation	CV	0.033	0.037	0.072

Table 13.Procedure 1. Scenario 3

Seventh Inspection				
New illegal rate	$\lambda_j\mu_j(6)$	258.803	271.279	137.5
Assets allocation	A_j7	9	8	3
# of ships to be inspected	d_j7	36	40	18
# of illegal ships found	K_j7	6	6	3
Updated Beta Distribution				
Alpha	$\alpha_j(7)$	671.70	478.27	146.94
Beta	$\beta_j(7)$	1679.30	867.40	446.81
Updated mean fraction of illegal ships	$\mu_j(7)$	0.29	0.36	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(7)$	0.009	0.013	0.018
Coefficient of variation	CV	0.033	0.037	0.072
Eigth Inspection				
New illegal rate	$\lambda_j\mu_j(7)$	257.137	266.559	136.111
Assets allocation	A_j8	9	8	3
# of ships to be inspected	d_j8	36	40	18
# of illegal ships found	K_j8	6	8	5
Updated Beta Distribution				
Alpha	$\alpha_j(8)$	677.70	486.27	151.94
Beta	$\beta_j(8)$	1709.30	899.40	459.81
Updated mean fraction of illegal ships	$\mu_j(8)$	0.28	0.35	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(8)$	0.0092	0.0128	0.0175
Coefficient of variation	CV	0.032	0.037	0.070
Ninth Inspection				
New illegal rate	$\lambda_j\mu_j(8)$	255.522	263.195	136.601
Assets allocation	A_j9	9	8	3
# of ships to be inspected	d_j9	36	40	18
# of illegal ships found	K_j9	8	7	6
Updated Beta Distribution				
Alpha	$\alpha_j(9)$	685.70	493.27	157.94
Beta	$\beta_j(9)$	1737.30	932.40	471.81
Updated mean fraction of illegal ships	$\mu_j(9)$	0.28	0.35	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(9)$	0.009	0.013	0.017
Coefficient of variation	CV	0.032	0.036	0.068
Tenth Inspection				
New illegal rate	$\lambda_j\mu_j(9)$	254.697	259.493	137.937
Assets allocation	A_j10	9	10	3
# of ships to be inspected	d_j10	36	50	18
# of illegal ships found	K_j10	7	8	4
Updated Beta Distribution				

Table 13.Procedure 1. Scenario 3

Alpha	$\alpha_j(10)$	692.70	501.27	161.94
Beta	$\beta_j(10)$	1766.30	974.40	485.81
Updated mean fraction of illegal ships	$\mu_j(10)$	0.28	0.34	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(10)$	0.009	0.012	0.017
Coefficient of variation	CV	0.032	0.036	0.068
Eleventh Inspection				
New illegal rate	$\lambda_j\mu_j(10)$	253.53	254.766	137.5
Assets allocation	A_j11	9	10	3
# of ships to be inspected	d_j11	36	50	18
# of illegal ships found	K_j11	11	8	4
Updated Beta Distribution				
Alpha	$\alpha_j(11)$	703.70	509.27	165.94
Beta	$\beta_j(11)$	1791.30	1016.40	499.81
Updated mean fraction of illegal ships	$\mu_j(11)$	0.28	0.33	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(11)$	0.009	0.012	0.017
Coefficient of variation	CV	0.032	0.036	0.101
Twelfth Inspection				
New illegal rate	$\lambda_j\mu_j(11)$	253.84	250.35	137.087
Assets allocation	A_j12	9	10	3
# of ships to be inspected	d_j12	36	50	18
# of illegal ships found	K_j12	9	14	4
Updated Beta Distribution				
Alpha	$\alpha_j(12)$	712.70	523.27	169.94
Beta	$\beta_j(12)$	1818.30	1052.40	513.81
Updated fraction of illegal ships	$\mu_j(12)$	0.28	0.33	0.25
Updated S.D. fraction of illegal ships	$\sigma_j(12)$	0.00894	0.01186	0.01652
Coefficient of variation	CV	0.032	0.036	0.101

Table 13.Procedure 1. Scenario 3

OBJECTIVE: MAXIMIZE NUMBER OF ILLEGAL SHIPS FOUND

			Area 1	Area 2	Area 3
		95% ODDS RATIO BOUNDS			
		LOWER	0.26	0.07	0.62
Total Assets (n)	20	95	1.78	0.55	2.60
Real X_j (j=1,2,3)	0.2				
Variable description		**Symbol**	**Area 1**	**Area 2**	**Area 3**
Area information					
Ship arrival rate		λ_j	900	750	550
Assets inspection capacity		Cap_J	4	5	6
Initial mean fraction of illegal ships		$\mu_j(0)$	0.28	0.34	0.23
Initial S.D. fraction of illegal ships		$\sigma_j(0)$	0.028	0.034	0.023
Initial Beta parameters		$\alpha_j(0)$	71.72	65.66	76.77
Alpha		$\beta_j(0)$	184.42	127.46	257.01
Beta					
First Inspection					
Illegal ship rate		$\lambda_j\mu_j(0)$	252	255	127
Initial assets allocation		$A_j(1)$	9	8	3
# of ships to be inspected		$d_j(1)$	36	40	18
Expected number of illegal ships		E[Kj(1)]	7.2	8.0	3.6
# of illegal ships found		$K_j(1)$	7	7	5
Posterior Odds Ratio		$P(X_j>\mu_j(0))/P(Xj<\mu_j(0))$	0.51	0.21	1.15
Updated mean fraction of illegal ships		$\mu_j(1)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships		$\sigma_j(1)$	0.03	0.034	0.023
Updated Beta parameters					
Alpha		$\alpha_j(1)$	71.72	65.66	76.77
Beta		$\beta_j(1)$	184.42	127.46	257.01
Updated illegal rate		$\lambda_j\mu_j(1)$	252	255	127
Seeond Inspeetion					
New assets allocation		A_j2	9	8	3
# of ships to be inspected		d_j2	36	40	18
Expected number of illegal ships		E[Kj(2)]	7.2	8.0	3.6
# of illegal ships found		K_j2	9	8	5
Posterior Odds Ratio		$P(X_j>\mu_j(1))/P(Xj<\mu_j(1))$	0.77	0.27	1.15
Updated mean fraction of illegal ships		$\mu_j(2)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships		$\sigma_j(2)$	0.028	0.034	0.019
Updated Beta parameters					
Alpha		$\alpha_j(2)$	71.72	65.66	76.77
Beta		$\beta_j(2)$	184.42	127.46	257.01
New illegal rate		$\lambda_j\mu_j(2)$	252	255	127

Table 13.Procedure 1. Scenario 3

Third Inspection				
New assets allocation	A_j3	9	8	3
# of ships to be inspected	d_j3	36	40	18
Expected value of illegal ships	E[Kj(3)]	7.2	8.0	3.6
# of illegal ships found	$K_j(3)$	7	7	2
Posterior Odds Ratio	$P(X_j>\mu_j(2))/P(Xj<\mu_j(2))$	0.51	0.21	0.62
Updated mean fraction of illegal ships	$\mu_j(3)$	0.28	0.34	0.23
Updated S.D. fraction of illegal ships	$\sigma_j(3)$	0.028	0.034	0.023
Updated Beta parameters				
Alpha	$\alpha_j(3)$	71.72	65.66	76.77
Beta	$\beta_j(3)$	184.42	127.46	257.01
New illegal rate	$\lambda_j\mu_j(3)$	252	255	127
Fourth Inspection				
New assets allocation	$A_{j(}4)$	9	8	3
# of ships to be inspected	$d_{j(}4)$	36	40	18
Expected value of illegal ships	E[Kj(4)]	7.2	8.0	3.6
# of illegal ships found	$K_{j(}4)$	5	7	10
Posterior Odds Ratio	$P(X_j>\mu_j(3))/P(Xj<\mu_j(3))$	0.33	0.21	3.21
Updated mean fraction of illegal ships	$\mu_j(4)$	0.28	0.34	0.56
Updated S.D. fraction of illegal ships	$\sigma_j(4)$	0.03	0.03	0.28
Update Beta Distribution				
Alpha	$\alpha_j(4)$	71.72	65.66	1.22
Beta	$\beta_j(4)$	184.42	127.46	0.98
New illegal rate	$\lambda_j\mu_j(4)$	252	255	306
Fifth Inspection				
New assets allocation	$A_{j(}5)$	8	6	6
# of ships to be inspected	$d_{j(}5)$	32	30	36
Expected value of illegal ships	E[Kj(5)]	6.4	6.0	14.4
# of illegal ships found	$K_{j(}5)$	6	4	13
Posterior Odds Ratio	$P(X_j>\mu_j(4))/P(Xj<\mu_j(4))$	0.52	0.23	0.01
Updated mean fraction of illegal ships	$\mu_j(5)$	0.28	0.34	0.36
Updated S.D. fraction of illegal ships	$\sigma_j(5)$	0.03	0.03	0.18
Update Beta Distribution				
Alpha	$\alpha_j(5)$	71.72	65.66	2.19
Beta	$\beta_j(5)$	184.42	127.46	3.88
New illegal rate	$\lambda_j\mu_j(5)$	252	255	199

Table 13.Procedure 1. Scenario 3

CV REDUCTION

t	CV Area 1	CV Area 2	CV Area 3
1	0.033	0.037	0.077
2	0.033	0.037	0.076
3	0.039	0.046	0.086
4	0.033	0.037	0.075
5	0.033	0.037	0.073
6	0.033	0.037	0.072
7	0.033	0.037	0.072
8	0.032	0.037	0.070
9	0.032	0.036	0.068
10	0.032	0.036	0.068
11	0.032	0.036	0.101
12	0.032	0.036	0.101

MEAN

t	Mean Area 1	Mean Area 2	Mean Area 3
0	0.3	0.4	0.25
1	0.30	0.39	0.26
2	0.29	0.38	0.25
3	0.29	0.38	0.25
4	0.29	0.37	0.25
5	0.29	0.37	0.25
6	0.29	0.36	0.25
7	0.29	0.36	0.25
8	0.28	0.35	0.25
9	0.28	0.35	0.25
10	0.28	0.34	0.25
11	0.28	0.33	0.25
12	0.28	0.28	0.25

Table 13.Procedure 1. Scenario 3

INSPECTIONS MISSIONS RESULTS SCENARIO: 3

ASSETS

INSPECTION	Area 1	Area 2	Area 3
1ST	9.0	8.0	3.0
2ND	9.0	8.0	3.0
3RD	9.0	8.0	3.0
4TH	9.0	8.0	3.0
5TH	8.0	6.0	6.0

POSTERIOR ODDS RATIO

INSPECTION	Area 1	Area 2	Area 3
1ST	0.507	0.212	1.148
2ND	0.774	0.271	1.148
3RD	0.51	0.21	0.62
4TH	0.327	0.212	3.213
5TH	0.52	0.23	0.01

MEAN

INSPECTION	Area 1	Area 2	Area 3
1ST	0.280	0.340	0.230
2ND	0.280	0.340	0.230
3RD	0.280	0.340	0.230
4TH	0.280	0.340	0.556
5TH	0.280	0.340	0.361

EXPECTED NUMBER OF ILLEGAL SHIPS

INSPECTION	Area 1	Area 2	Area 3	Total
1ST	7	7	5	19
2ND	9	8	5	22
3RD	7	8	4	19
4TH	7	8	4	19
5TH	6	6	14	27
				106

ILLEGAL SHIPS FOUND

INSPECTION	Area 1	Area 2	Area 3	Total
1ST	7	7	5	19
2ND	9	8	5	22
3RD	7	7	2	16
4TH	5	7	10	22
5TH	6	4	13	23
				102

Table 13.Procedure 1. Scenario 3

APPENDIX B. CUMULATIVE RIGHT TAIL BINOMIAL PROBABILITIES

$d_j(t+1) = 12$

			Cumulative Right Tail Binomial Probabilities					
				$K_j(t+1)$ Values				
μ_j	0	1	2	3	4	5	6	7
0.02	1	0.215	0.023	0.002	0.000	0.000	0.000	0.000
0.04	1	0.387	0.081	0.011	0.001	0.000	0.000	0.000
0.06	1	0.524	0.180	0.032	0.004	0.000	0.000	0.000
0.08	1	0.632	0.249	0.065	0.012	0.002	0.000	0.000
0.1	1	0.718	0.341	0.111	0.026	0.004	0.001	0.000
0.12	1	0.784	0.431	0.167	0.048	0.009	0.001	0.000
0.14	1	0.836	0.517	0.230	0.075	0.018	0.003	0.000
0.16	1	0.877	0.595	0.299	0.111	0.031	0.006	0.001
0.18	1	0.908	0.664	0.370	0.155	0.049	0.012	0.002
0.2	1	0.931	0.725	0.442	0.205	0.073	0.019	0.004
0.22	1	0.949	0.778	0.511	0.261	0.102	0.030	0.007
0.24	1	0.963	0.822	0.578	0.320	0.138	0.045	0.011
0.26	1	0.973	0.859	0.640	0.382	0.179	0.065	0.018
0.28	1	0.981	0.890	0.898	0.445	0.225	0.089	0.027
0.3	1	0.986	0.915	0.747	0.507	0.276	0.118	0.039

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   Naval Postgraduate School
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